



Delaware Department of Education

Teaching and Learning Branch

Formative Assessment Probes

Math Investigations

Grade 5

Prepared by:
Crystal Lancour, Education Associate
Mathematics K–8

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A Case for Formative Assessment

Delaware has adopted the definition of **Response to Intervention** as published by the National Association of State Directors of Special Education (2005):

RTI is the practice of providing high-quality instruction and intervention matched to student need, monitoring progress frequently to make decisions about change in instruction or goals and applying child response data to important educational decisions. RTI should be applied to decisions in general, remedial and special education, creating a well-integrated system of instruction/intervention guided by child outcome data.

Delaware's three-tiered RTI model is an efficient and effective model that emphasizes high-quality differentiated instruction based on meeting the needs of each student using data collected during formative assessment, diagnostic assessment, and summative assessment.

Successful implementation of **Response to Intervention** requires Delaware educators to:

1. Believe that you can effectively teach all students.
2. Use a research-based, scientifically validated core curriculum with fidelity.
3. Use assessment data to inform your instructional decisions.
4. Use a problem-solving method to make decisions within a multi-tier model of service delivery.
5. Use research-based, scientifically validated interventions matched to student need.

In response to these five principles, the Delaware mathematics community designed the Response to Intervention (RTI) project. Formative assessment is the cornerstone of the Delaware RTI model based on the strongest possible research-based evidence connecting the use of diagnostic assessment and descriptive feedback to improved student achievement (Black, P., & William, D., 1998; and Crooks, 1988). We adopted the Chief Council of State School Officers' definition and five critical attributes of formative assessment listed below.

Formative assessment is a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes.

Effective use of formative assessment requires attention to the following attributes:

- **Learning Progressions:** Learning progressions contain clearly articulated sub-goals of the ultimate learning goal.
- **Learning Goals:** Learning goals and criteria for success are clearly identified and communicated to students.
- **Descriptive Feedback:** Students receive evidence-based feedback linked to the intended instructional outcomes and criteria for success.
- **Self- and Peer-Assessment:** Substantial student self- and peer-assessment are important for providing students an opportunity to think meta-cognitively about their learning.
- **Collaboration:** Practitioners establish and support a classroom culture in which teachers and students are partners in learning.

(Developed and approved by the CCSSO Formative Assessment Advisory Group and Formative Assessment for Teachers and Students (FAST) SCASS, October 2006)

These formative assessment principles have been used to build curriculum-based templates that guide teachers through the process of identifying learning gaps and implementing interventions. The work is remedial in nature and designed to be integrated with acceleration strategies.

Formative Assessment: Delaware’s Vision of Response to Intervention

As much as possible, as we wrote this document, we tried to imagine what this kind of implementation of Response to Intervention would look like in classrooms. We tried to imagine formative assessment in all of its messiness, as well as its potential, in order to create a document that would provide practical support to teachers as well as vision.

Delaware’s RTI program is designed to occur within the classroom setting, using the curriculum materials that districts are already using. Teachers are supposed to analyze student work on pages that are part of the lessons themselves, and then use class time to intervene with students in need of Tier 1 intervention. While this may sound simple in conception, it is actually very complex and calls upon teachers to be experts in management, assessment, planning, and teaching.

Classroom Management and Classroom Culture

Tier 1 of RTI may require teachers to carefully observe students as they work, ask “in the moment” questions, and carry out interventions with individual, pairs, or small groups of students during class time. For this to be accomplished they need to establish a classroom culture which will allow this. Students need to learn to:

- Work productively by themselves and with others.
- Get, share, and return the materials they need.
- Ask and receive help from other students when the teacher is unavailable.
- Find something else productive to do when they are finished or stuck.

For help establishing such a culture, we recommend these resources:

- “Setting Up the Investigations Classroom” in the *Implementing Investigations* guides for each grade level. This section focuses on establishing a flexible and challenging learning environment for all students. Strategies include addressing classroom culture, effectively grouping students, managing manipulatives, and pacing instruction.
- *The First Six Weeks of School* by Paula Denton and Roxann Kriete.
- Getting Metacognition Out of the Closet
(<http://investigations.terc.edu/library/implementing/qa-1ed/metacognition.cfm>)

Assessment

In order to gain as much information about students as possible, teachers need to assess their students in many ways. Teachers need to:

1. Observe students as they work and take notes in an organized way.
2. Carefully analyze student work on the formative assessment checkpoints after class.
3. Think about student understanding and misunderstanding not just in terms of “getting it” or “not getting it,” but in specific language that describes what the student does or does not understand. In analyzing student work as part of this project, we found that most student mistakes or difficulties fell into one of these categories:
 - a. Lack of conceptual understanding;
 - b. Inefficient strategy;
 - c. Misusing or not using an important mathematical tool or representation;

- d. Difficulty with verbal or written communication;
- e. No connection to prior knowledge.

Recommended resources:

- The “ongoing assessment” section of each lesson in the curriculum.
- Teacher Note: Computational Fluency and Place Value in the *Implementing Investigations* guides at each grade level.
- Teacher Note: Representations and Contexts for Mathematical Work in the *Implementing Investigations* guides at each grade level.
- How Do I Organize My Observations?
(http://investigations.terc.edu/library/implementing/ga-1ed/organize_observations.cfm)

Planning

In order to create lessons and interventions that help all students learn, teachers need to carefully plan their lessons. While management of materials and students is extremely important, **it is also extremely important that teachers plan the mathematical focus of their lessons**. For every activity, class discussion, or intervention, teachers need to figure out ahead of time what mathematical skills, concepts, or strategies they are trying to develop in their students. Much of this information can be gathered from:

- Reading the “About the Mathematics” section in each of the *Investigations* units.
- Reading the lessons carefully.
- Doing the math ahead of time and anticipating student responses.
- Reading the “Teacher’s Notes” in each *Investigations* unit.

Class discussions are often not planned carefully ahead of time, and the students who are most affected by this lack of planning tend to be struggling students. Class discussions that are carefully focused follow clear trains of thought and draw clear connections among different strategies or ideas will extend the learning of all students in important ways.

Recommended resources for planning class discussions are:

- How Can I Help Special Needs Students Feel Included in Math Discussions?
(http://investigations.terc.edu/library/implementing/ga-1ed/special_needs_class_disc.cfm)
- “Inclusive Math Communities” (http://investigations.terc.edu/library/implementing/ga-1ed/inclusive_communities.cfm)
- The Teacher Note: Discussing Mathematical Ideas in the *Implementing Investigations* guides at each grade level.

Likewise, when teachers are intervening with students, they need to carefully think about the primary goal(s) for their intervention. Not all interventions are designed for the same purpose. Some are designed to help the student who is lagging behind in conceptual understanding of a topic. Others are written to improve computation, communication, representation, and connections.

- **Conceptual Leap** – This type of intervention is designed to prompt students to develop or clarify a particular concept or generalization. For example, a second-grade teacher might design an intervention to prompt a student to develop a generalization about breaking 2-digit numbers into 10s and 1s. Or, a teacher may need to diagnose where

the child is in a specific learning trajectory and use an intervention based on the child's concept knowledge.

- **Computation Efficiency** – This type of intervention is designed to help students understand and use more efficient computation strategies. For example, a third-grade teacher might design an intervention to prompt students who are drawing pictures to solve multiplication problems, to begin using skip counting instead. A primary teacher might look at a student who is counting all and design an intervention for counting on.
- **Use of Tools or Models** – This type of intervention is designed to help students learn to use a particular tool or model to solve problems. Students in need of this intervention may not understand how a model relates to the mathematics. A fourth-grade teacher might design an intervention to help students understand and begin to use an array model to solve multiplication problems with 2-digit numbers. A primary teacher may design an intervention to help students make sense of a tens frame.
- **Communication** – This category includes both interventions designed to help students read and understand what a problem is asking and interventions designed to help students clearly communicate their thinking both verbally and in writing. It also includes carefully structuring a lesson so that vocabulary that is unknown (like campfire) does not get in the way of solving a problem. A fifth-grade teacher might design an intervention to help students read and make sense of multi-step word problems. Another fifth-grade teacher might design an intervention to help students record their work on multi-step word problems in a way that was clear and concise.
- **Prior Knowledge or Experiences** – This category of intervention is designed to prompt students to make connections among work they did earlier in the year or in a previous year and a current mathematical concept or task. For example, a fourth-grade teacher might design an intervention in which students divided a paper brownie into fractional parts and labeled the parts as they had in third grade. The purpose of this intervention would be to remind students of what they did know about fractions in order to prepare them for the fourth-grade fraction tasks.

Teaching

In thinking about the teaching of interventions, we asked ourselves two basic questions:

- When would interventions happen?
- What would interventions look like?

We decided that the most convenient time for interventions to occur would be during math workshop. Therefore, many of the interventions that we wrote occur during math workshop. One of the explicit goals of math workshop in the *Investigations* curriculum is “to give teachers time to work one-on-one with small groups of students” (page 12 of the *Implementing Investigations* guides at each grade level). When we felt that an intervention should not wait until the next math workshop, we suggested that it occur in one of the next few lessons, during an independent work time. Other teachers have started a lesson with the whole class and find time to pull a small group together while the rest of the class is working in small groups. Some schools are finding daily “intervention” time and sharing that time between English language arts and mathematics. While some students are receiving Tier 2 intervention outside of class, others can receive Tier 1 intervention or extension activities.

What the teaching of interventions should look like was a broader question for us. Most of the interventions we wrote are written as small-group activities, facilitated by the teacher, and can be implemented as such. However, we are aware that teachers may not always be available to

lead small-group lessons, and sometimes the number of students needing an intervention might be more than a “small group.” We also recognize that teachers also may not find it beneficial to gather the same students for small-group lessons too frequently. We do not want these students to become stigmatized in the eyes of their peers or themselves, and we do not want these students to become overly dependent upon adult help. We imagined that most interventions could be accomplished in one of four forms, with different forms being preferable under different circumstances. The four forms are:

- **Small-group lessons with the teacher** – This form is suggested most frequently in our intervention probes. In this form of intervention, the teacher meets target group of students for 10 to 25 minutes during the regular math class to provide interventions.
- **Strategic partnering of students** – In this form of intervention, a teacher carefully chooses particular students to collaborate as partners based on the belief that in working together one or both children will help the other child develop a particular skill or understanding. For example, a teacher might choose to have a student who had difficulty using a number line to solve subtraction problems work with a student who is able to use a number line to solve subtraction problems with the goal that the student who is having difficulty using the number line will learn from the other child. We recommend that teachers who use strategic partnering tell students in clear, but tactful language, what they hope the students will each contribute to and gain from the partnership. Strategic partnering could be especially helpful for students who lack vocabulary or background knowledge.
- **Whole-class interventions** – In this form of intervention, a teacher may modify a task for the whole class or plan a whole-class discussion in a particular way, based upon information learned from analyzing student work. For example, a teacher who realized that many students in her class have difficulty with the initial tasks in the fourth-grade fraction unit might revisit a third-grade fraction lesson with her whole class in order to activate the prior knowledge of all of the students in the class. Another teacher whose class was not clearly recording their work might take a lesson for addition strategies and focus on how each strategy could be recorded clearly.
- **Modification of tasks for particular students** – In this form of intervention, a teacher modifies a task or substitutes a different task for particular students based upon the learning goals for those students. For example, a third-grade teacher may have three students in her class who are just learning to add numbers by place. The teacher might give those three students the same word problems that she gives the other students in her class but changes the numbers in the word problems to make them 2-digit numbers instead of 3-digit numbers. That same teacher could also have the three students play a second-grade game during a Tier 1 math time as a way to improve their number sense and understanding of place value. This modification or substitution of a task is a good way to foster independence and perseverance among students who often ask for help. It can also be an effective homework strategy. By modifying a task or substituting a different task, a teacher can provide homework that can be done independently by the child and is focused on a targeted skill or concept.

We recommend the following resources for further ideas on implementing the types of interventions suggested above:

- Chapter 7: Working with the Range of Learners” in the *Implementing Investigations* guides at each grade level.
- “I’m Wondering About the Frequency and Importance of Choice Time?” (http://investigations.terc.edu/library/implementing/ga-1ed/choice_time.cfm)

- “How Can I Help Special Needs Students Feel Included in Math Discussions?”
(http://investigations.terc.edu/library/implementing/qa-1ed/special_needs_class_disc.cfm)
- “Math Menu: A Variation on Choice Time that Helps Teachers Support the Range of Learners” (http://investigations.terc.edu/library/implementing/qa-1ed/math_menu.cfm)
- “Strategies for Special Needs Students”
(http://investigations.terc.edu/library/implementing/qa-1ed/special_needs_strategies.cfm)
- “How do I enrich math class to challenge gifted students?”
(http://investigations.terc.edu/library/implementing/qa-led/enrich_for_gift_students.cfm)

Progress Monitoring

Selected Checkpoints from the *Investigations* curriculum will serve as Progress Monitors for your Tier 1 students. Good teaching in the regular classroom along with a system to track students periodically will help all students meet the standards in mathematics. Look over the packet of Checkpoints for your grade level along with the big ideas tested in the Universal Screening Tool.

Formative assessment is used to monitor student achievement.

- It is not graded but instead used to provide information about how each student in your classroom is doing based on the standards. Imagine giving each of these checkpoints and walking around the room as students are working on them. The best part is that all students are doing the checkpoints as a part of an instructional day, so asking students to do checkpoints as you watch them work is not a new idea and children will not feel as though they are being graded.
- The teacher has a rare opportunity to watch students do math and ...
 - *ask them questions to clarify the problem,*
 - *stop them to redirect their thinking, and*
 - *probe the students who need additional help.*
- Then, by looking at the student work, the teacher can ...
 - *make a plan to have some students work in small groups based on your observations,*
 - *make a plan for whole-group instruction,*
 - *look to future lessons and use this information to teach what the students need.*
- The teacher directions are in a separate packet from the *Investigations* curriculum, but lists the checkpoints for formative assessment as well as three components.
 - *Rationale:* This section has been included by classroom teachers in the pilot study to describe why the particular problem is useful for assessing the content and what teachers might learn about their students by looking at their work.
 - *Instructions for Implementation:* This section has been included to provide teachers with guidance for implementing the checkpoints.
 - *“In the Moment Questions and Interventions:* Again, the teachers in the pilot tested some interventions and found some to be very workable. The teachers decided whether the intervention was to be large group, small group, individual, or sometimes whole class. There are also questions that can be asked during “real time” of implementation to gather more information about students’ thinking.
- The Universal Screening Tool includes a spreadsheet that groups questions into categories based on Numeric Reasoning. We have correlated these checkpoints to the screener, as well. The “Connections to *Investigations*” table will be provided in a separate packet to share some information for how to address areas of need in the screening tool through *Investigations* curriculum. It also correlates these needs to specific Checkpoints for which the concepts may be formatively assessed again.
- In each grade level, there are certain manipulatives that are standard for your grade level. In almost every case, students should be given access to manipulatives when solving these problems. A rule of thumb for using manipulatives: Make available to each student what is usually given during the learning of the skill.

Please direct your questions to:

- Sarah Ryan, University of Delaware, sryan@udel.edu
- Crystal Lancour, Department of Education, clancour@doe.k12.de.us
- Diana Roscoe, Department of Education, droscoe@doe.k12.de.us

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Colonial School District

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Sincerely,

Crystal Lancour & Diana Roscoe, Delaware Department of Education

Grade 5

Assessment Checkpoints for Grade 5, Unit 1

| Checkpoint | Task and Session | <i>Investigations</i> Benchmarks (Unit Goals) | Delaware Prioritized GLEs* |
|--------------|---|--|---|
| Checkpoint 1 | Multiplying to Make 18 and 180 (Session 1.4) | (1) Find the factors of a number. | Determine factor pairs that make up a given number. (Grade 4 essential GLE) Develop an understanding of the commutative and associative properties of whole number multiplication as tools for solving problems. (Grade 4 essential GLE) |
| Checkpoint 2 | Assessment Checklist: Number Puzzles (Session 1.5) | (1) Find the factors of a number. | Determine factor pairs that make up a given number. (Grade 4 essential GLE) Describe numbers according to characteristics, such as even, odds, factors, multiples, and squares. (I) |
| Checkpoint 3 | Multiplying Two Ways (Session 2.2) | (2) Solve multiplication problems efficiently. | Multiply and divide by large numbers (2-digit by 2-digit) and show why the operation works. (E) Use multiplication clusters to build mental math strategies. (E) |
| Checkpoint 4 | Division Problems (Session 3.3) | (3) Solve division problems with 1-digit and 2-digit divisors. | Multiply and divide by large numbers (2-digit by 2-digit) and show why the operation works. (E) |

Grade 5, Unit 1, Session 1.4– Checkpoint 1: Multiplying to Make 18 and 180

Teacher’s Guide, page 49, and SAB, page 12

Rationale: Investigation 1 focuses on reasoning about numbers and their factors. This formative assessment checkpoint offers teachers the opportunity to assess how easily their students can find the factors a number (benchmark 1).

Directions: Before giving this assessment, read the teacher note, Using Mathematical Vocabulary and Finding Prime Factors, on Teacher’s Guide (TG) pages 153-155 and the Dialogue Box, Multiplying with More Than Two Numbers, on TG pages 181-182. Follow the directions for session 1.4. As students work on SAB page 12, observe and ask in the moment questions. Pay attention to:

- How students use one solution to find others.
- Whether students can systematically find multiple combinations that make 18 and 180.
- Whether students use multiplying by 10 to find combinations for 180.

After class, analyze the student papers more carefully.

Learning Goals:

1. Finding the factors of a number (benchmark 1).
2. Using known multiplication combinations to find other multiplication combinations.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|--|
| Student finds only a few multiplication combinations and/or the student does not use the multiplication combinations he/she has found for 18 to make multiplication combinations for 18. | <p>Do you think you have found all of the multiplication combinations with two factors? Why or why not?</p> <p>Do you think you could use one of these multiplication combinations that you have found for 18 to find another combination for 18 or a combination for 180?</p> <p>I see you wrote $6 \times 3 = 18$. Could you use that combination to help you find a similar combination for 180?</p> | <p>These students may need some explicit instruction on how to use one multiplication combination to generate others. They may also need explicit instruction on how to generate factors of a number systematically.</p> <p>During math workshop in sessions 1.5 and 1.6, pair these students with students who are able to use one combination to generate others and/or who are able to generate factors systematically.</p> |
| Student easily finds factors of 18 and 180 and finds many multiplication combinations with two and three factors. | <p>How did you find so many multiplication combinations for 18 and 180?</p> <p>Do you think you have found all of the combinations with two factors? With three factors? How do you know?</p> | <p>Follow the extensions suggestions on TG page 51. Ask these students to try to find all of the possible combinations for 180 with two, three, four, and more factors. If they complete this task, have them do the same with 240.</p> |

Grade 5, Unit 1, Session 1.5 – Checkpoint 2: Assessment Checklist: Number Puzzles

Teacher’s Guide, page 54, and M33-M34

Rationale: Investigation 1 focuses on reasoning about numbers and their factors. This checkpoint offers teachers the opportunity to assess how well their students can reason about numbers and find the factors of a number (benchmark 1).

Directions: Before giving this assessment, read the Teacher Note, Using Mathematical Vocabulary, on TG page 153 and the Dialogue Box, Solving a Number Puzzle, on TG pages 178-180. Follow the directions for session 1.5, including the directions on TG page 54 for assessing students as they solve number puzzles 13 and 14. Fill out the assessment checklist for each student as you observe him/her.

Learning Goals:

1. Finding the factors of a number (benchmark 1).
2. Using properties (even, odd, prime, square) and relationships (factor, multiple) of numbers to solve problems.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|--|--|
| <p>Student has difficulty generating the factors of a number.</p> <p>The student may incorrectly identify some numbers as factors that are not factors. The student may find only a few factors of 42, 70, or 60.</p> <p>The student may confuse the word factor with the word multiple and find multiples instead.</p> | <p>Why don’t you write down the factors that you have found so far so that you don’t forget them?</p> <p>Do you think you have found all of the factors of 42? Why or why not?</p> <p>Do you think using a 300 chart would help you?</p> | <p>Monitor these students for the rest of the investigation. In sessions 1.6 and 1.7, pair these students with students who are a little better than they are at finding factors and solving number puzzles. In session 1.7, follow the intervention suggestion on TG page 64.</p> <p>These students probably are not fluent with their multiplication and division facts. Meet with these students individually to look over their lists of multiplication combinations that they still need to master (SAB page 10). Make a plan with each of these students for how they are going to learn all of the multiplication combinations that they do not yet know in the next two months. Send home multiplication games and more homework sheets like SAB page 9 for them to use to practice.</p> |
| <p>Student has difficulty using all of the clues to find the answers.</p> <p>The student may not understand all of the vocabulary, or the student may have difficulty working</p> | <p>Are there any of these words that you are not sure of?</p> <p>Which clue would you like to start with? Which clue would you like to use next?</p> <p>Can you think of a way to keep track of your possible answers?</p> | <p>During sessions 1.6 and 1.7, pair these students with students who are a little better at finding factors and solving number puzzles than they are, and monitor these students throughout the rest of this unit. These students may have difficulty with vocabulary and/or multi-step problems.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|--|
| with several pieces of information at one time. | | Make a math vocabulary chart and post it in the classroom where all students can see and refer to it. |
| Students easily solve the number puzzles. They find all the factors of a number easily and integrate the information from all four clues quickly and easily. | <p>How did you solve that puzzle so quickly? What did you think about first? Then what did you do? Then what did you do?</p> <p>Did you solve the second puzzle in a similar way or differently?</p> | When these students finish all of the number puzzles and SAB pages 12, 15, 19, and 20, pair them with other similar students and ask them to write their own number puzzles. |

Grade 5, Unit 1, Session 2.2 – Checkpoint 3: Multiplying Two Ways

Teacher’s Guide, page 84, and SAB, page 28

Rationale: Investigation 2 focuses on helping students develop efficient strategies for 2-digit by 2-digit multiplication (benchmark 2). Students studied 2-digit by 2-digit multiplication in unit 8 of fourth grade. By the end of this investigation, students should be able to solve these problems in one of three ways:

1. By breaking the numbers apart by addition (or place value);
2. By changing one number to make an easier problem; or
3. By creating an equivalent problem.

See TG pages 161-162 for an explanation of each of these strategies.

By the end of the year, students should be able to understand and use all three strategies. This formative assessment checkpoint allows teachers to assess how students are progressing toward these goals.

Directions: Read the Teacher Notes, Representing Multiplication with Arrays, Multiplication Strategies, and Developing Computation Strategies, on TG pages 156-162 and TG pages 171-173. Follow the directions for session 2.2, **making time for students to work individually on SAB page 28 (the daily practice page and assessment checkpoint) at the end of the math period or later in the day.** As students work on SAB page 28, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Solve a 2-digit by 2-digit multiplication problem efficiently and accurately, using one of the three strategies described above (benchmark 2).
2. Record their solution clearly.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|--|
| Student uses an inefficient strategy (e.g., pictures, tallies, or cubes or repeated addition). | Can you think of a faster way to solve this problem? Earlier today, we talked about the strategies of breaking one number apart by place value or breaking both numbers apart by place value, and using arrays to keep track of the parts of the problem. Did you understand that conversation? Do you think you could use the strategy of breaking apart the numbers to solve the problem 26×11 ? How would you start? If 26×11 appears too hard, ask the student to solve 14×5 . | Make sure that the multiplication strategies that have been discussed so far in whole-class discussions are clearly recorded (both with and without arrays) on chart paper and posted where all students can see them. Plan future discussions of strategies carefully. Make sure the discussions have a clear focus, that the students who share speak clearly and loudly, and that other students have a chance to comment and ask questions about the strategies that are shared. Also, record student strategies clearly on the board or on chart paper. (See dialogue box on TG pages 83-84.) |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|--|
| | | <p>It is important that all students begin using the strategy of breaking one or both numbers apart by place value. For students who are having difficulty with this strategy, adjust the numbers in the problems (including the problems on their homework) to make them smaller and more manageable (i.e., make one number in the problem a single-digit number or a number in the low teens).</p> <p>Pair these students with students who have a better understanding of the strategy of breaking apart numbers by place value and/or meet with a small group of these students during math workshop in sessions 2.4 and 2.5 to work on the “cluster problems” or the “problems about teams.” Follow the other intervention suggestions throughout the investigation, as needed.</p> |
| <p>Student makes a major error in trying to solve the problem. (For example, student breaks apart the numbers in ways that do not make sense.)</p> | <p>Can you tell me how you solved this problem?</p> <p>Do you have a way to figure out if your answer is reasonable? How could you figure out if your answer was reasonable or not?</p> <p>I think you are trying to use the strategy of breaking apart the numbers by place value, but you left out part of the problem. Let’s try using an array to figure out what part you left out.</p> <p>Let’s draw an array. What part of the array have you solved so far? What part(s) do you still need to figure out?</p> <p>Or</p> <p>Can you think of a story problem to go with 26×19? What part of the story problem have you solved so far? What part(s) do you still need to solve?</p> <p>If the student is still confused, try offering a problem with smaller numbers to solve.</p> <p>Try solving 23×1 or 14×5.</p> | <p>The “in-the-moment” questions at the left are meant to help the teacher assess the extent of the student’s strengths and misconceptions. Does the student have a way to figure out if his/her answer is reasonable? Can the student detect his/her own error? Can the student use an array or story context to help him/her keep track of the parts of the problem? Can the student accurately solve a problem with smaller numbers?</p> <p>After assessing the student’s strengths and misconceptions, follow the intervention suggestions in the box above.</p> |
| <p>Student has the correct answer but has not clearly</p> | <p>Can you tell me how you solved this problem?</p> | <p>If many students are not writing clear enough explanations, try this whole-class intervention. Identify</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|---|
| recorded his/her thinking. | <p>Do you think that what you just told me matches what put down on your paper?</p> <p>What is different about what you told me and what you wrote?</p> <p>What could you add or change so that someone could read it and know exactly how you solved the problem?</p> | <p>two or three student papers with clear explanations. If possible, use papers that show different strategies. Show these papers to the class using an overhead projector or Elmo. After you give students time to read each explanation, ask, “What makes this a clear explanation?” “Is there anything else we could add to or change in this explanation to make it even clearer?”</p> <p>Also, continue to model clear recording of solutions on the board and on chart paper.</p> |
| Student solves both problems accurately and efficiently, demonstrating a clear understanding of at least two of the three strategies listed above. | | <p>These students are ready to solve 3-digit by 2-digit multiplication problems. They may benefit by being paired together to work on these problems. Also, look for other extension suggestions in the book to meet the needs of these students.</p> |

Grade 5, Unit 1, Session 3.3 – Checkpoint 4: Division Problems

Teacher’s Guide, page 127, and SAB, page 53

Rationale: Investigation 3 focuses on division. Students are expected to learn to interpret and solve division problems with 2-digit divisors (benchmark 3), to use multiples of 10 to solve division problems, and to make sense of remainders. This formative assessment checkpoint allows teachers to assess whether their students are meeting these expectations and, if not, what they need to work on.

Directions: Read the Teacher Notes, Division Notation, About Cluster Problems, The Relationship Between Multiplication and Division, Division Strategies, and Developing Computation Strategies That Make Sense, on TG pages 163-165 and TG pages 169-173. Follow the directions for session 3.3. As students work on SAB page 53, observe and ask in-the-moment questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Interpret and solve division problems with 2-digit divisors.
2. Create a story problem to go with a division problem.
3. Use multiples of 10 to solve division problems.
4. Make sense of remainders.
5. Record their solution clearly

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|---|
| <p>Student does not use multiples of 10 to solve any of the division problems.</p> <p>Student uses a counting strategy (e.g., pictures, tallies, or cubes) or repeated addition to solve the problem, without multiplying the divisor by 10 or a multiple of 10 as a first step.</p> | <p>Can you think of a faster way to solve this problem?</p> <p>Do you think the answer will be more than 10 or less than 10? Why do you think that?</p> <p>After 10 shelves are filled, how many more books will still need to be put away? How could you figure out how many shelves it would take to put the rest of the books away? (If a student uses repeated addition or repeated subtraction to figure out how many shelves it will take to put the rest of the books away, it is okay.)</p> | <p>Note: Students do not always make a connection between finding patterns on multiple towers and solving division problems. The work with the multiple towers is supposed to help students become more familiar and flexible with multiplying numbers by 10, 20, 30, and other multiples of 10. After making multiple towers, students are expected to use their ability to multiply numbers by 10 (and multiples of 10) as the first step in solving division problems.</p> <p>Monitor these students throughout the rest of the investigation. Make sure that they are attentive and involved during the whole-class discussion during session 3.4, in which efficient division strategies are clearly explained and clearly recorded for the whole class to see. Make charts showing the different division strategies that the students share and post the charts where students</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|--|
| | | can easily see and refer to them. Explicitly discuss with these students the connection between multiple towers and division problems. If these students continue to have difficulty, try adjusting the numbers in their problems to make them smaller (although usually make the dividend at least 10 times greater than the divisor). |
| Student has difficulty writing division word problems when given a bare number problem. The students’ word problem may be incomplete, or it may not be a division problem. | <p>Can you explain how your word problem is an example of 525 divided by 21?</p> <p>What do you know about division word problems?</p> <p>Can you think of a word problem that fits 25 divided by 5?</p> | Meet with these students in a small group to discuss the elements of a division problem. Explicitly explain that a division problem involves dividing a quantity into equal groups. Explain that in some division problems you are given the quantity and the number of groups, and you have to figure out the number in each group. In other division problems, you know the quantity and the number in each group, and you have to figure out the number of groups. Then practice writing division word problems together. |
| Student is not dealing with remainders in a way that makes sense. | Can you explain your answer to me? If the student does not catch his/her own mistake, use descriptive feedback to point out what does not make sense about the way the student is using remainders. Ask the student to try fixing the mistake. | Monitor these students throughout the rest of the investigation. Ask these students to share their work with a partner to make sure that it makes sense before they turn it in. Also, give descriptive feedback as needed and ask the students to correct their mistakes. |
| Student has the correct answer but has not clearly recorded his/her thinking. | <p>Can you tell me how you solved this problem?</p> <p>Do you think that what you just told me matches what put down on your paper?</p> <p>What is different about what you told me and what you wrote?</p> <p>What could you add or change so that someone could read it and know exactly how you solved the problem?</p> | Model very clear recording of division solutions during the class discussion in session 3.4 (TG page 134). Record students’ strategies clearly on chart paper, and post them where students can easily see and refer to them. Ask students to show their work to a peer to see if it is clear enough before they turn it in. |

Assessment Checkpoints for Grade 5, Unit 3

| Checkpoint | Task and Session | <i>Investigations</i> Benchmarks (Unit Goals) | Delaware Prioritized GLEs |
|--------------|--|--|---|
| Checkpoint 1 | Addition and Subtraction Problems (Session 1.3) | <i>Assesses prior knowledge that is needed to efficiently add and subtract numbers in the thousands.</i> | Develop and use strategies to estimate the results of operations on whole numbers. (E) |
| Checkpoint 2 | More Subtraction Problems (Session 2.3) | (2) Solve subtraction problems accurately and efficiently, choosing from a variety of strategies. | Add and subtract larger numbers and explain how the operation works. (Grade 4 important GLE) Add and subtract decimals using money models (Grade 4 important GLE) |
| Checkpoint 3 | Using Stadium Data (Session 3.2) | (2) Solve subtraction problems accurately and efficiently, choosing from a variety of strategies. | Apply more than one operation to solve a word problem. (E) Select and use appropriate methods and tools for computing (mental math, estimation, calculators, paper and pencil) depending on the context and nature of the problem. (E) |

Grade 5, Unit 3, Session 1.3 – Checkpoint 1: Addition and Subtraction Problems

Teacher Guide, page 31, and SAB, page 1

Rationale: Investigation 1 focuses on deepening students’ understanding of place value up to 10,000 and adding and subtracting numbers up to 10,000. In fourth grade, students **should have learned** to add and subtract multiples of 10, 100, and 1,000 to and from numbers in the hundreds and thousands. Students also **should have learned** to add and subtract 3-digit numbers efficiently using the same strategies that are focused on in this unit. (See TG pages 183-185 for a detailed description of each of these strategies.)

- Subtracting in parts
- Adding up and subtracting back
- Changing the numbers
- Subtracting by place

Adding and subtracting multiples of 10, 100, and 1,000 is an essential prerequisite for developing efficient addition and subtraction strategies. It is also an important indicator of how well students understand our number system. This formative assessment checkpoint allows teachers to assess whether their students have this essential concept and skill so they can identify and work intensely with any students who do not.

Directions: Leave time at the end of session 1.1 or the beginning of session 1.2 for students to do SAB page 1. Observe the students as they work to see whether or not students can add and subtract using mental math. Ask students to try solving these problems without using the 10,000 chart.

Learning Goals: Add and subtract multiples of 10, 100, and 1,000 fluently.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/ Activities |
|--|--|---|
| <p>Student is unable to correctly answer the addition and subtraction problems.</p> <p>Student is dependent upon the 10,000 chart or the standard algorithm to solve these problems and cannot solve them mentally.</p> <p>The student can add or subtract multiples of 10 and/or 100 but not multiples of 1000.</p> | <p>Explain to me how you found your answer. Could you do this problem in your head?</p> <p>What happens when you add 10 (or 100 or 1,000) to a number? Which digits change? Which digits stay the same? Why does this happen? What happens when you add 20 (or 200 or 2,000)? Which digits change? Which digits stay the same? Why? What happens when you subtract these numbers?</p> <p>What patterns did you see when you use the 10,000 chart to add 10 to a number (or 100 or 1000)?</p> | <p>During session 1.3, while other students are playing How Many Steps to 10,000?, work with a small group of students who have difficulty adding and subtracting multiples of 10,100, or 1,000.</p> <p>Do the 10-Minute Math activity Practicing Place Value with these students. Start with a number between 500 and 1,000 and ask students to add and subtract a few multiples of 10 and then 100. After each problem ask, “Which digits changed? Which did not? Why?”</p> <p>Continue to do this 10-Minute Math with the whole class, whenever the TG says to, and more frequently if possible. Make sure all students are actively engaged and involved.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/ Activities |
|-----------------------------------|---------------------------|---|
| | | <p>Instead of having these students play Close to 1,000 in sessions 1.3 and 1.4, make up more problems like the ones on SAB page 1 for them to solve.</p> <p>Modify the numbers in the homework for session 1.3, and if needed, the related problems on SAB pages 13-15 to make them smaller. Try to keep the same patterns in the related problem sets. Follow the intervention suggestions on TG page 49 for working with students on the Related Problems, if necessary.</p> |

Grade 5, Unit 3, Session 2.3 – Checkpoint 2: More Subtraction Problems

Teacher’s Guide, page 74, and SAB, page 33

Rationale: In Investigation 2, students solve subtraction problems using multiple strategies and tools. At this point, most students should have selected favored strategies and should be working to become more accurate and efficient. However, some students are still unable to solve subtraction problems correctly and/or are unable to use a strategy efficiently.

Directions: Before the lesson begins, read pages 119-123 (Subtraction Strategies) in the Teacher’s Guide. Follow the directions for session 2.3, leaving enough time for the daily practice problem, which is the checkpoint. As students work on SAB page 33, walk around and talk to them about their strategies. Look for students making computational errors and probe their thinking to determine which subtraction strategy they are attempting to use. As you see students struggling, use the “in-the-moment” questions to determine the nature of their misunderstandings. Identify students with critical misunderstandings. These students should be grouped together during math workshop. Students who are answering correctly but who are working inefficiently should work in a separate group.

Learning Goals: Solve subtraction problems accurately and efficiently, choosing from a variety of strategies (benchmark 2).

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/ Activities |
|---|--|--|
| <p>Student is unable to solve subtraction problems correctly.</p> <p>Student uses a promising strategy but is unable to complete all the parts.</p> <p>Student uses a counting up or counting back strategy but not efficiently.</p> <p>Student can find the correct answer but cannot explain the answer in terms of place value.</p> <p>Student uses the standard algorithm incorrectly or without understanding.</p> | <p>How big do you think this answer should be?</p> <p>What strategy are you using to solve this problem?</p> <p>Why did you choose this strategy?</p> <p>How do you know when you are finished?</p> <p>Can you use place value to break the numbers apart?</p> <p>Can you subtract multiples of 10 and 100?</p> <p>Where did you start counting up (or back)?</p> <p>Can you solve this problem a different way?</p> <p>Can you do parts of this problem in your head?</p> | <p>Small Group Intervention during session 2.4: While the whole class is working in math workshop, target 4 or 5 students who demonstrate similar difficulties with subtracting correctly. Explain to these students why you selected them and what you want them to learn from working with you. In place of workshop activity 2A, guide the students through a mini-lesson on subtracting on the open number line using counting on or counting back. Then, have them solve problems from Practicing Subtraction, SAB page 23, as you observe and offer help as needed.</p> <p>Then, present the students with their work from the checkpoint, SAB page 33. Ask students to re-do the checkpoint problems. As they finish, tell students to move to activity 2C, Distance Problems.</p> <p>Next, work with a second group of students who are not working efficiently. Ask them to solve problems from Activity 2B using a number line. Encourage students to make larger jumps on the number line.</p> |

Grade 5, Unit 3, Session 3.2 – Checkpoint 3: Using Stadium Data

Teacher's Guide, page 95, and SAB, pages 51-52

Rationale: Throughout this unit, students have practiced subtraction using a variety of strategies and tools (Benchmark 2). These strategies, as described in the teacher notes on page 119, include:

- Subtraction in parts
- Adding up or subtracting back
- Changing the numbers to solve a “friendlier” subtraction problem
- Subtracting by place

By the third investigation, some students may have become more fluent with multiple strategies while other students are still experimenting and practicing a preferred strategy. As the numbers become larger, students are encouraged to focus on solving problems by using what they know about place value. These problems are designed to help students capitalize on their understanding of how to add and subtract multiples of hundreds and thousands. Although students are encouraged to use multiple strategies, number lines may be useful in helping students to move towards efficiency and conceptual fluency (see teachers notes on page 118).

Directions: Before the lesson begins, read pages 118-119 in the Teacher's Guide. While preparing to launch the lesson, the teacher may anticipate that some students could encounter difficulty reading and interpreting the stadium data chart. Think about how to provide support for the class either as a whole group or in small groups to make sense of the chart and to carefully interpret the data. After reminding students that they will be solving problems that involve addition and subtraction, the teacher should walk around the room, observing what strategy students are using and finding out which students are struggling.

Learning Goals:

1. Interpret and solve multi-step problems.
2. Add and subtract multiples of 100 and 1,000.
3. Solve addition and subtraction problems with large numbers by focusing on the place value of the digits.
4. Solve subtraction problems accurately and efficiently (benchmark 2).
5. Record work clearly and concisely.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|--|
| <p>Student is unable to recognize whether the problem involves addition or subtraction.</p> <p>Student uses one of the four strategies (see above) but is unable to solve the problems correctly.</p> <p>Student is unable to describe why the strategy they are using is effective.</p> <p>Student uses the standard algorithm incorrectly and without understanding.</p> <p>Student can find the correct answer but cannot explain the answer in terms of place value.</p> | <p>What are you trying to find out in this problem?</p> <p>Can you tell me <i>about</i> how many people are in the stadium so far?</p> <p>Can you make a sketch of the stadium and show me how many people are in it so far?</p> <p>Can you do some part of this problem in your head?</p> <p>How can you use place value to solve the problem?</p> | <p>During Lesson 3.2: After students work for 20 minutes, select 5-6 students who are struggling to come and work with the teacher.</p> <ul style="list-style-type: none"> Explain to the small group that you would like them to practice mental math models. Using “friendlier numbers,” ask them to solve a series of questions such as: $1,500 - 900$? $15,000 - 9,000$? $25,000 - 10,000$? As students answer, record the problem and the answer on a chart. Once all students have tried using mental math strategies, ask students if it is true that $15,000 - 10,000$ is like $15 - 10$? Move from these mental math problems back to the stadium problems. Change the stadium numbers to “friendlier” numbers (multiples of 1,000) in order for the students to re-do the problems. |
| <p>Tool Use:</p> <ul style="list-style-type: none"> Student uses number line or 10,000 chart ineffectively. <p>Efficiency:</p> <ul style="list-style-type: none"> Student uses counting up or counting back strategies without using “jumps” of multiples of 100s or 1,000s. Student is overly reliant on the 10,000 chart and demonstrates reluctance to use mental math strategies. | <p>Can you use friendlier numbers to make this problem easier to solve?</p> <p>Can you solve this problem using fewer jumps?</p> <p>Can you show me a different way to solve this problem?</p> <p>Can you solve this problem without using the 10,000 chart?</p> <p>Can you do parts of this problem in your head?</p> | <p>During session 3.3: Lead the whole-class discussion of strategies at the beginning of session 3.3. During math workshop, return to students who required intervention during session 3.2. Check in to see how students are doing.</p> <ul style="list-style-type: none"> If needed, remind them about the strategies discussed at the beginning of class. Use two samples of student work from your book (or from another classmate) to help them review two effective strategies counting up on the number line and subtracting parts by place (see page 99 strategies shown). Ask students to talk to each other to try to figure out how each of these students was thinking. Ask the students to try one of these strategies for their own use. |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|--|
| | | <ul style="list-style-type: none"> • In a small group, work on the first problem from SAB page 59, Rock On! together. Start by asking students if they could solve this problem in their head. If not, ask them if they can change one of the numbers so that they can. Elicit from the students what they plan to try. • Allow students to try their strategy. • Once students are meeting success, send them back into their group for the next workshop activity. <p>Repeat the intervention with a second group of students who are answering questions correctly but who are not efficient and/or flexible.</p> <p>Have students share their strategies and compare them. Name the strategies and discuss how efficiently students used each one. (Did they use chunks of large numbers? Did their strategies require just a few steps or many?)</p> |
| <p>Communication:</p> <ul style="list-style-type: none"> • Student fails to use clear and concise notation to describe the thinking process. | <p>What can you add to your work to make it easier for the teachers and other students to understand?</p> | <p>At the beginning or end of each workshop, showcase student work exemplars and describe specific reasons why the students’ work is an example of organization and communication.</p> |

Assessment Checkpoints for Grade 5, Unit 4

| Checkpoint | Task and Session | Investigations Benchmarks (Unit Goals) | Delaware Prioritized GLEs |
|--------------|--|--|---|
| Checkpoint 1 | What Do You Already Know? (Session before 1.1) | Assesses assumed prior knowledge: <ul style="list-style-type: none"> • Ability to identify fractional parts of a group. • Ability to read, write, and interpret fraction notation. • Knowledge of equivalent fractions. • Knowledge of combinations of fractions equal to 1. | Develop understanding of fractions as parts of unit wholes, as part of a collection, as locations on number lines, and as division of whole numbers. (E) |
| Checkpoint 2 | Solving Problems with Fraction and Percents (Session 1.5) | (1) Use fraction-percent equivalents to solve problems about the percentage of a quantity. | Generate and connect equivalent forms of benchmark fractions, decimals, and percents. (E) Develop the meaning of percent as a ratio of a number out of 100. (I) |
| Checkpoint 3 | Which Is Greater? (Session 2.2) | (2) Order fractions with like and unlike denominators. | Compare and order fractions using physical models, pictures, and number lines. (Grade 4 essential GLE) Generate and connect equivalent forms of benchmark fractions, decimals, and percents. (E) |
| Checkpoint 4 | Assessment Checklist: Adding Fractions (Session 3.3) | (3) Add fractions through reasoning about fraction equivalents and relationships. | Add and subtract benchmark fractions and fractions with common denominators using physical models. (E) Add and subtract fractions with unlike denominators and use physical models to justify your answer. (Grade 6 essential GLE) |

Grade 5, Unit 1, Session Before 1.1 – Checkpoint 1: What Do You Already Know?

Teacher’s Guide, page 25, and SAB, pages 1-2

Rationale: This formative assessment checkpoint was designed by the authors as a formative assessment. It is supposed to be given in the very first session of this unit. However, **we recommend that teachers give this assessment to students 4 weeks before the unit begins**, so that they have time to work with students who do not demonstrate the prerequisite skills and understandings before the unit begins. Students are expected to enter fifth grade already knowing a lot about fractions (see TG page 10 for a detailed list of what they should have learned in grades 3 and 4). This assessment offers teachers the opportunity to assess whether students have learned as much about fractions and percents as the curriculum assumes they know.

Directions: Give this assessment about 4 weeks before you plan to begin teaching Unit 4. This will allow you to work with students who are lacking some essential prior knowledge of fractions, for a few weeks before you begin teaching unit 4 to the whole class. As students work on SAB page 1-2, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Identify fractional parts of a group (problems 1, 5, and 6).
2. Demonstrate understanding of and correct use of the numerator and denominator (all problems).
3. Demonstrate knowledge of equivalent fractions (problems 2, 5, and 7).
4. Understand that 100% refers to all or the whole, and 50% refers to one half of the whole (problem 4).

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|--|---|
| Students who have difficulty with any of the problems except problem 4. | <p>Tell me about the answer that you wrote here.</p> <p>Why did you decide to write this fraction?</p> <p>Why did you decide to put this number on the top?</p> <p>Why did you decide to put this number on the bottom?</p> <p><i>(These questions are not designed to help the child figure out the correct answer but to help the teacher understand the misconceptions the child may have.)</i></p> | <p>The classroom teacher or another teacher will need to work regularly with these students on fractions before the class begins work on Unit 4. As the teacher works with these students, he/she should regularly ask students to explain their thinking in order to monitor student progress and identify misconceptions.</p> <p>Here is a list of lessons from previous units that might be appropriate for these students. The teacher will need to decide which lessons will be most beneficial.</p> <ul style="list-style-type: none"> • Making Fraction Sets, 3rd grade, unit 7, session 1.2 and 1.3 • Sharing 7 Brownies, 3rd grade, unit 7, session 1.4 • The Fraction Cookie Game, 3rd grade, unit 7, session 2.2 |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|---|
| | | <ul style="list-style-type: none"> • Making Half-Yellow Designs, 3rd grade, unit 7, session 2.4. (If you use color tiles instead of pattern blocks for this task and ask students to make a rectangle that is $\frac{1}{2}$ yellow, the task may be easier for the students, and you can use the designs to explore equivalent fractions. i.e., a rectangle with 6 yellow color tiles out of a total of 12 invites a conversation about whether $\frac{6}{12}$ is equivalent to $\frac{1}{2}$.) • Fourth grade, Unit 6, Investigation 1. (All 7 lessons.) <p>Note: It is appropriate to re-teach or revisit the lessons and worksheets that students used in previous years. Students are unlikely to remember the exact problems they worked on in years before, and they benefit from playing the same games more than once.</p> |
| Student has difficulty with the percent problem. | <p>Have you ever heard anyone use the terms 100% or 50%?</p> <p>Do you remember what they were talking about?</p> <p>Do you have a guess about what 100% might mean or what 50% might mean?</p> | <p>These students do not require any intervention. Percents were not introduced in fourth grade. Explicitly explain what these terms mean when you teach the lessons about percents. Monitor these students during the percent lessons.</p> |

Grade 5, Unit 4, Session 1.5 – Checkpoint 2: Solving Problems with Fractions and Percents

Teacher’s Guide, page 49, and SAB 15, M11

Rationale: This investigation focuses on understanding the meaning of fractions and percents. This checkpoint offers teachers the opportunity to assess how well students can *apply* what they have just learned about percents and fraction-percent equivalents to solve word problems about fractions and percents (benchmark 1).

Directions: Before giving this assessment, read the Teacher Notes, About Teaching Fractions and Percents Together and Visualizing Fractions and Percents, on TG pages 145-149. Follow the directions for session 1.5. Ask “in-the-moment” questions and fill out the assessment checklist for each student as you observe him/her working on the problems on SAB pages 15-16. Analyze each student’s work more carefully after class.

Learning Goals:

1. Know basic fraction-percent equivalents.
2. Name part of a group as a fraction.
3. Use knowledge of percents and fraction-percent equivalents to find percentages of a group (benchmark 1).

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|---|
| <p>Student has difficulty with problems involving basic fraction-percent equivalents like $1/2 = 50\%$, $1/3 = 33\frac{1}{3}\%$, $3/4 = 75\%$, and $1/5 = 20\%$. (The ones in problems 1-3.)</p> <p>Difficulties with these problems may indicate:</p> <ul style="list-style-type: none"> • The student is having difficulty remembering basic fraction-percent equivalents • The student has difficulty naming a part of a group as a fraction <p>And/or</p> | <p>For a student who seems stuck:</p> <ul style="list-style-type: none"> • Why don’t you sketch what is happening in the problem. Do you what fraction of the class helped with the canned food drive? Can knowing the fraction help you to figure out the percent? • Do you know what fraction $66\frac{2}{3}\%$ is equal to? <p>For a student who already has an incorrect answer:</p> <ul style="list-style-type: none"> • Can you tell me how you thought about this problem? How did you figure out your answer? These questions will help you to detect any misconceptions the student has about fractions or percents. | <p>The teacher will need to figure out which kind of difficulty (in bullets on the left) each student who is having trouble with these problems is having.</p> <p>Students in the first category need to revisit the work they did with grids. Students in the second category need more experience with the 10-Minute Math activity, Guess My Rule. Students in all categories need to meet with the teacher to work on more of these types of problems. The teacher may decide to work with all of these students together or in smaller groups according to the type of difficulty they are having. <i>The teacher will have to make up some similar problems for the students to work on.</i></p> <p>It is important that these students who are having difficulty not miss any of the upcoming activities about fractions and percents in Investigation 2. Therefore, the teacher should meet with these students outside of the regular math class to work with them, or the teacher should spend an extra day giving the whole class more fraction-percent word</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|----------------------------------|--|
| <ul style="list-style-type: none"> The students have difficulty connecting percents to fractions when the percents and fractions are not colored on a grid. | | <p>problems. On this extra day, the teacher would work with small groups of students who are having difficulty while the rest of the class works on fraction-percent problems individually or with partners. If possible, the teacher should make up a set of problems with basic fraction-percent equivalents for the students having difficulty and more challenging fraction-percent equivalents for the rest of the class.</p> |

Grade 5, Unit 4, Session 2.2 – Checkpoint 3: Which Is Greater?

Teacher's Guide, page 66, and SAB, page 21

Rationale: Investigation 2 focuses on developing strategies for comparing and ordering fractions with like and unlike denominators (benchmark 2). In this investigation, students are expected to develop and use several *different* strategies for comparing and ordering fractions such as:

- Using fraction equivalents they know—example $\frac{3}{5} = \frac{6}{10}$. $\frac{6}{10}$ is less than $\frac{7}{10}$. Therefore, $\frac{3}{5}$ is less than $\frac{7}{10}$.
- Using fraction-percent equivalents—example $\frac{3}{8} = 37\frac{1}{2}\%$. $\frac{1}{3} = 33\frac{1}{3}\%$. Therefore, $\frac{3}{8}$ is greater than $\frac{1}{3}$.
- Using landmarks such as $\frac{1}{2}$, 1, and 2—example $\frac{4}{3}$ is greater than 1. $\frac{3}{4}$ is less than 1. Therefore, $\frac{4}{3}$ is greater than $\frac{3}{4}$.
- Using 4×6 or 5×12 rectangles—example $\frac{3}{8}$ of a 4×6 rectangle is 9 square units. $\frac{1}{3}$ of a 4×6 rectangle is 8 square units. 9 is greater than 8, so $\frac{3}{8}$ is greater than $\frac{1}{3}$.

This formative assessment checkpoint allows teachers to assess:

1. Whether each student in the class has some way to compare the fractions on the assessment checkpoint;
2. Which students use a range of strategies for comparing fractions and which students use only one or two strategies, and
3. Which strategies the class as a whole tends to use to compare fractions and which strategies the teacher needs to encourage the class as a whole to develop.

Directions: Read the Teacher Notes, Strategies for Comparing Fractions, on TG pages 152-53 and the excerpts of what *students might say* in sessions 2.2-2.4 and make sure that you understand these examples of student reasoning. Follow the directions for session 2.2, **including having students put away their Percent Equivalent Strips** (page 64). As students work, observe and ask “in-the-moment” questions. Use the ongoing assessment questions on page 66 as a guide for what to look for as you observe students, and follow the intervention suggests on page 67 if needed. After class, analyze the student responses on SAB page 21 more carefully.

Learning Goals:

1. Compare fractions using the strategies described above (benchmark 2).
2. Explain reasoning clearly.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|---|
| <p>Student does not use any of the strategies described above to compare fractions. The student might only draw pictures, like those on TG page 67, or show no work. (Drawing freehand pictures is different from using 4 x 6 or 5 x 12 rectangles. Using the rectangles is considered an appropriate strategy.)</p> | <p>Can you explain how you figured out which fraction is greater?</p> <p>These fractions are very close in size. It is hard for me to tell just from your drawing which fraction is greater. Can you think of any other ways to prove that $7/10$ is greater than $3/5$?</p> <p>Could you use equivalent fractions to prove which fraction is greater? Do you know how many tenths $3/5$ is equal to? Could that help you to compare the fractions?</p> <p>Could you use percent equivalents to figure out which fraction is greater?</p> <p>Could you use rectangles to figure out which fraction is greater? Which size rectangle would you use? Why?</p> <p>See page 67 for other ideas of questions to ask.</p> | <p>Note: Students who fall into this category either do not have a strong understanding of fractions, or they do not have clear ways to express their reasoning, or both. It is important that teachers do not teach these students just one strategy, like using fraction-percent equivalents, to solve these types of problems. These students need to be exposed to and expected to practice at least three ways to compare fractions. This unit focuses on the strategy of fraction-percent equivalents, but that is only because the students were expected to have learned the other strategies in fourth grade.</p> <p>During session 2.3, monitor these students as they complete the Fraction and Percent Equivalents chart. (Follow the intervention suggestions on TG page 72 for students who have difficulty filling in their chart.)</p> <p>During session 2.3, when the rest of the class plays the In Between game, these students should:</p> <ul style="list-style-type: none"> • Finish their Fraction and Percent Equivalents chart • Repeat the activity “Putting Fractions in Order” (see TG page 75 for details) AND/OR • Play the In Between Game using fewer cards (e.g., playing with just the halves, fourths, eighths, and thirds will encourage students to use equivalent fractions to compare the halves, fourths, and eighths and use percents to place the thirds.) <p>In the class discussion during session 2.4, explicitly name the four strategies for comparing fractions that are described above. Before discussing problem 8, discuss problem 1. Have students explain how they solved it, and you name the strategy they used. If not all strategies were used, show the students how they could have used other strategies as well. Chart how each strategy could be used to solve problem 1 and post the chart.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|---|---|
| | | Meet with these students as a small group during math workshop in session 2.4, 2.5, or 2.6. Play the In Between game with them or work on some of the Which is Greater? problems. |
| Student uses only one strategy (the same strategy to solve all four problems) and/or makes mistakes when solving one or more of the problems. | <p>Can you tell me how you solved this problem?</p> <p>Do you have any other way to figure which fraction is greater?</p> <p>If the student does not, ask these questions one at a time, and give the student plenty of think time.</p> <ul style="list-style-type: none"> • Could you use equivalent fractions to help you figure out which fraction is greater? • Could you use percents? • Could you use rectangles? Which size rectangle would you choose? • Could you figure out how far each fraction is from 1 or from $\frac{1}{2}$? | <p>The “in-the-moment” questions at the left are meant to help the teacher further assess what strategies the student has for comparing fractions and to offer the student a chance to correct his/her mistake.</p> <p>If during questioning students do not demonstrate an ability to use more than one strategy accurately to compare fractions, follow the intervention suggestions written in the box above.</p> |
| The student has strategies for solving the problems but does not record his/her thinking clearly. | <p>Can you tell me how you solved this problem?</p> <p>Do you think that what you just told me matches what you put down on your paper?</p> <p>What is different about what you told me and what you wrote?</p> <p>What could you add or change so that someone could read it and know exactly how you solved the problem?</p> | Clearly explaining one’s reasoning about fractions is different from clearly recording the steps it took to solve a multiplication problem. You will need to model the language you expect students to use when explaining their thinking, both verbally and in writing. You will need to model this language with the whole class and individually with students who still have difficulty. Make and post a chart showing what it would look like to use each of the four strategies described above to solve problem 1 on SAB page 21. Remind students to refer to the chart when they have difficulty explaining their thinking. |

Grade 5, Unit 4, Session 3.3 – Checkpoint 4: Assessment Checklist: Adding Fractions

Teacher’s Guide, page 109; SAB, page 46; and M24

Rationale: Investigation 3 focuses on adding fractions through reasoning about fraction equivalents and relationships (benchmark 3). At this point in fifth grade, students have been exposed to a variety of strategies for adding fractions, such as using clock fractions, percent equivalents, 4 x 6 and 5 x 12 rectangles, equivalent fractions, and just knowing familiar combinations of fractions. This formative assessment checkpoint allows the teacher to assess:

1. Whether all students are able to use at least one strategy to solve these problems;
2. What strategies individual students use to solve the problems;
3. What strategies the class as a whole tends to use and which strategies the class needs more practice with; and
4. If any individuals or groups of students have any misconceptions or are making particular mistakes when trying to add fractions.

Directions: Read the Teacher Notes, Adding and Subtracting Fractions, on TG pages 160-161 and End of the Unit Assessment, Problem 3, on TG page 167-169. Follow the directions for session 3.3. As students work on SAB page 46, observe and ask “in-the-moment” questions, and record what you observe on the Assessment Checklist. After class, analyze the student papers more carefully. Although the TG says to only focus on student responses to problems 1 and 2, analyze all of the student responses for this assessment checkpoint.

Learning Goals:

1. Add fractions through reasoning about fraction equivalents and relationships (benchmark 3).
2. Explain and record their thinking clearly.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|--|--|
| <p>Student does not have strategies for accurately solving many of the problems. The student may appear stuck for a long time on particular problems, misuse a strategy, or give incorrect answers to several problems.</p> | <p>Follow the intervention suggestions on TG page 110.</p> | <p>Monitor these students during sessions 3.4-3.6 as they fill in a Fraction Tracks game board—notice patterns on the Fraction Tracks, order fraction cards, and learn to play the game Fraction Tracks. Pair them with students who are a little more flexible than they are in adding fractions and finding equivalents. Make sure these students are attentive and involved in the class discussion during session 3.5. Follow the intervention suggestion on TG page 125 during session 3.6 when students begin playing Fraction Tracks.</p> <p>During math workshop in sessions 3.7-3.9, follow the suggestions on TG page 132 for deciding how these students should spend their time. These students should probably spend at least some of their math workshop time playing Roll Around the Clock.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|---|
| | | Meet with these students in a small group during one of the math workshop sessions to help them get started on the Fraction Problems pages. If the students have difficulty thinking of ways to solve the problems, follow the intervention suggestions on TG page 134. |
| Student answers all of the problems correctly but only uses one strategy for adding fractions. | <p>Can you explain to me how you solved this problem?</p> <p>You seem to be very good at using the _____ strategy to add fractions. Can you think of another way to add these fractions? (Give the student plenty of wait time to think of another strategy.)</p> <p>Could you use clock fractions to solve this problem?</p> <p>Could you use a 4 x 6 or 5 x 12 rectangle to solve this problem?</p> | <p>If you did not have a chance to ask these students the “in-the-moment” questions at the left, find time the next day to ask them these questions or write the students notes on their papers acknowledging their correct answers, but also asking them to think of another way to solve 2 of the problems.</p> <p>If the students cannot think of another way to solve the problems, find time at the end of session 3.3 or during session 3.4 to meet with them to discuss how to use both the clocks and the rectangles to solve one or two of the problems.</p> <p>Observe these students during session 3.6 as they begin playing Fraction Tracks. Ask yourself the Ongoing Assessment questions on page 124. Follow the intervention suggestions on page 125 if needed.</p> |
| Student quickly and easily solves all of the questions and demonstrates verbally or in writing an ability to use a variety of different strategies to add fractions. | Can you tell me how you solved this problem? Can you think of another way to add these fractions? | Pair these students with other students who have a strong understanding of several different fraction models (clocks, percent equivalents, and rectangles). Follow the extension suggestions in sessions 3.6 and 3.7. During math workshop in sessions 3.7 -3.9, these students should spend all their time playing Fraction Track and Solving Fraction Problems. If they have more time, they could write their own fraction problems. |

Assessment Checkpoints for Grade 5, Unit 6

| Checkpoint | Task and Session | <i>Investigations</i> Benchmarks (Unit Goals) | Delaware Prioritized GLEs |
|-------------------|---|--|--|
| Checkpoint 1 | Decimals on Hundredths and Thousandths Grids (Session 1.2) | (1) Read, write, and interpret decimal fractions to thousandths. | Use multiple methods and models to convert decimals to fractions and fractions to decimals. (E) Develop the meaning of percent as a ratio of a number out of 100. (E) |
| Checkpoint 2 | The Decimal Trail (Session 1.5) | (2) Order decimals to thousandths. | Use multiple methods and models to compare decimals. (E) |
| Checkpoint 3 | Decimal Problems (Session 2.4) | (3) Add decimal fractions through reasoning about place value, equivalents, and representations. | Add and subtract decimals using models. (E) |

Grade 5, Unit 6, Session 1.2 – Checkpoint 1: Decimals on Hundredths and Thousandths Grids

Teacher’s Guide, pages 34-36, and SAB, pages 10-13

Rationale: Investigation 1 of this unit focuses on reading, writing, and interpreting fractions to thousandths (benchmark 1) and ordering fractions to thousandths (benchmark 2). Students were introduced to decimals in fourth grade. In fourth grade, they learned to read, write, and interpret decimals in tenths and hundredths and to compare decimals (in tenths and hundredths) to decide which was greater. Much of this unit should be building on what students already know. This formative assessment checkpoint allows teachers to assess early in the investigation whether students have prior knowledge of decimals and/or whether they are connecting their new learning about thousandths to what they already learned in fourth grade. It also allows teachers to assess whether students can connect their understanding of decimals to fractions and percents.

Directions: Read the Teacher Notes, About Teaching Decimals, Fractions, and Percents Together and Extending Place Value to Thousandths and Beyond, on TG pages 121-124. Follow the directions for session 1.2. As students work on SAB pages 10-13, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Represent decimal fractions as part of an area (correctly shade the grids).
2. Read and write decimal fractions in the tens, hundredths, and thousandths (benchmark 1).
3. Identify decimal, fraction, and percent equivalents.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|---|--|
| Student has difficulty shading some of the grids correctly. | <p>Can you read this decimal for me?</p> <p>How did you decide how much of this grid to shade in?</p> <p>Do you know what the 1 in 0.125 means? Did you shade that part of the decimal on your grid? Show me.</p> <p>Do you know what the 2 in 0.125 means? Did you shade that part of the decimal on your grid? Show me.</p> <p>Do you know what the 5 in 0.125 means? Did you shade that part of the decimal on your grid? Show me.</p> <p>Use descriptive feedback to explain to the student the mistakes you see him/her making. Explain to the student why the mistakes are wrong and try to correct any misconceptions you think the student has.</p> | <p>Monitor these students over the next several days. Pair them with students whose understanding of decimals is a little stronger than theirs. Make hundredths and thousandths grids available during the next few lessons and encourage these students to shade in grids to help them order decimals.</p> <p>Follow the intervention suggestions for sessions 1.3 and 1.4 on TG pages 42 and 46.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|--|
| | <p>If needed, change the decimals in some of the problems to make them easier. For example, give the student only problems with tenths and hundredths.</p> | |
| <p>Student has thinking of decimal, fraction, and percent equivalents.</p> | <p>Do you know what you are supposed to do on this part of the paper?</p> <p>Do you know how to say this decimal? Say it for me.</p> <p>Can you think of a fraction that is equivalent to 0.3?</p> <p>Do you think the name of this decimal (three-tenths) can help you think of a fraction?</p> <p>Do you know how to write three-tenths as a fraction?</p> <p>Do you know any other fractions this decimal is equivalent to? Do you know how many hundredths, $\frac{3}{10}$ is equal to? How many hundredths did you shade when you shaded in 0.3 on this chart?</p> <p>Do you know what percent this fraction is equivalent to?</p> <p>Remind the student of the work he/she did with percents in unit 4. Remind the student that percents are out of 100, so $\frac{30}{100} = 30\%$.</p> | <p>If a number of students are having difficulty with this, find time to give students one or two of these problems each day, for homework, for morning work, or for 5 minutes at the very beginning of math class. Students will be assessed on these types of problems again in session 1.5.</p> <p>Have these students (or all students) mark their number lines on SAB page 40 with equivalent fractions and percents for each of the decimals they place on their number lines.</p> |

Grade 5, Unit 6, Session 1.5 – Checkpoint 2: The Decimal Trail

Teacher’s Guide, page 55, and SAB, page 25

Rationale: Sessions 1.3-1.6 focus on having students order decimal fractions (benchmark 2). This formative assessment checkpoint allows teachers to assess how well their students can order fractions to the thousandths and whether they can connect their understanding of decimals to their understanding of percents.

Directions: Read the Teacher Note, Assessment, Comparing and Ordering Decimals, on TG pages 127-121 and the Dialogue Box, Putting Decimals in Order, on TG pages 138-139. Follow the directions for session 1.5, leaving time for the Daily Practice Problem and the assessment checkpoint at the end. As students work on SAB page 26, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully. Also, use any data you have collected from observing students playing In Between and Smaller to Larger, and from the Decimal Problems Assessment to decide how well your students are able to order fractions.

Learning Goals:

1. Order decimals to thousandths (benchmark 2).
2. Identify decimal and percent equivalents.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|---|--|
| <p>Student has difficulty getting started. The student may not understand the assignment.</p> | <p>What do you understand about this assignment? Can you read these decimals for me? Do you know why percents are written on this trail? Do you think you could use the percents to help you place some of these decimals on the trail? Can you find the decimal that is equivalent to 40%? How did you know this decimal is equivalent to 40%? Can you find the decimal that is equivalent to 50%? How did you know that this decimal is equivalent to 50%? Which decimal do you think goes between 70% and 80%? Why do you think that? Use descriptive feedback to explain to the student any misconceptions you hear and share correct information. If needed, change some of the decimals in the thousandths to decimals in the hundredths.</p> | <p>Use the answers to the questions at the left and other information that you know about these students to figure out whether they are having difficulty because they do not understand how to order decimals well, because they do not understand the connection between percents and decimals well, or because this context is new and different from other ordering of decimals they have done. If the problem is the context, no intervention is necessary. If the problem is a lack of understanding of the connection between decimals and percents, meet with these students during session 1.6. Ask them to do the Ordering Decimal activity from session 1.3, TG pages 40-41. Make copies of SAB page 17 for students to use. Ask students to choose 5 cards, place them on a number line, and write the percent equivalent below the decimal. Have students try helping each other as you observe. Offer help when needed, but also allow students enough time to try to figure out the placement and percent equivalents on their own.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|---|
| | | <p>If the problem is a lack of understanding about ordering decimals, meet with these students during session 1.6. Do the Ordering Decimal activity listed above, but leave out the part about listing the percent equivalents. Also, instead of having students <i>draw</i> 5 cards randomly, choose a selection of cards in the tenths, hundredths, and thousandths for the students to place. Make hundredths and thousandths grids available for students to use to order the decimals. After students order the decimals, helping each other when necessary, lead a discussion about how they decided where to place their decimals.</p> |
| <p>Student understands the assignment and is able to correctly place some of the decimals but places others incorrectly.</p> | <p>Point to both correctly and incorrectly placed decimals and ask,</p> <p>How did you decide to place this decimal here? Listen for patterns in the student’s thinking.</p> <p>Do you think you could use the percents to help you place some of these decimals on the trail?</p> <p>Do you think this decimal belongs close to the beginning of the trail or close to the end? Why?</p> <p>Do you think this decimal is greater than 50% or 1/2 or less than 50% or 1/2? Why do you think so?</p> <p>Use descriptive feedback to explain to the student any misconceptions you hear and share correct information.</p> <p>If needed, change some of the decimals in the thousandths to decimals in the hundredths.</p> | <p>If the student still has difficulty after the questioning, follow the intervention suggested above for students who have a lack of understanding about ordering decimals.</p> |

Grade 5, Unit 6, Session 2.4 – Checkpoint 3: Decimal Problems

Teacher’s Guide, pages 55, and SAB, page 52

Rationale: Investigation 2 focuses on teaching students to use reasoning about place value, equivalents, and representations. This formative assessment checkpoint allows teachers to assess how well their students are adding decimals and what strategies they are using. Teachers should work to move their students from coloring in actual grids to solve the problems, to other strategies like **adding by place**, **choosing combinations the students know and can add mentally**, and **mentally picturing the grids**.

Directions: Read the Teacher Notes, Adding Decimals, on TG pages 132-133 and End-of-Unit-Assessment, Problem 2, on TG pages 135-137 and the Dialogue Boxes, Adding Decimals: The Jeweler’s Gold and Student Strategies for Adding Decimals, on TG pages 142-146. Follow the directions for session 2.4. As students work on SAB page 52, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Add decimal fractions through reasoning about place value, equivalents, and representations (benchmark 3).
2. Record work clearly.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|---|
| Student has difficulty adding the decimals accurately. | <p>First ask the student about one of the easier problems, like number 1, 2, or 5.</p> <p>Tell me how you solved this problem. Do you remember the strategies for adding decimals that we discussed yesterday? Do you know what strategy you are using?</p> <p>Do you think your answer is reasonable? Why or why not?</p> <p>If the student does not catch his/her mistake, use descriptive feedback to explain it.</p> <p>If you shaded this decimal on a grid, do you know what the grid would look like?</p> <p>Do you think you could add these numbers by place? Could you add all of the whole numbers, then add the tenths, then add the hundredths?</p> | <p>Make a chart of the different strategies for adding decimals that your class has discussed so far and post it in the classroom where students can see and refer to it.</p> <p>Monitor these students during the math workshop activities during session 2.5 and offer help as needed. If necessary, have them work only with decimals in the tenths and hundredths to help them develop reliable strategies adding numbers.</p> <p>If these students are still having difficulty, meet with them during math workshop in session 2.6 or 2.7 to work on adding fractions.</p> |
| Student does not record work clearly. | Do you think your work is as clear as it could be? How could you make it clearer? | Model clear recording of strategies for adding decimals. |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|---|
| | Give the student descriptive feedback about any other aspect of the work that you would like him/her to notice or change. | Ask students to have a peer check their work to see if it is clear enough before they turn it in. |

Assessment Checkpoints for Grade 5, Unit 7

| Checkpoint | Task and Session | <i>Investigations</i> Benchmarks (Unit Goals) | Delaware Prioritized GLEs |
|--------------|---|--|---|
| Checkpoint 1 | Assessment Checklist: Equivalence in Multiplication (Session 1.1, 1.2, and 1.3) | (1) Explain why doubling one factor in a multiplication expression ($a \times b$) and dividing the other by 2 results in an equivalent expression. | Develop an understanding of the distributive property of whole number operations as a tool to solve problems. (I) Develop an understanding of the commutative and associative properties of whole numbers as tools to solve problems (Grade 4 essential GLE) |
| Checkpoint 2 | Multiplication: How Did I Solve It? (Session 2.1) | (2) Solve multiplication problems efficiently. | Multiply and divide by large numbers and show why the operation works. (E) |
| Checkpoint 3 | Division Practice (Session 3.2) | (3) Solve division problems efficiently. | Multiply and divide by large numbers and show why the operation works. (E) Use and apply various meanings of multiplication and division. (E) |

Grade 5, Unit 7, Session 1.1, 1.2, and 1.3 – Checkpoint 1: Assessment Checklist: Equivalence in Multiplication

Teacher’s Guide, page 30, and M15

Rationale: Investigation 1 focuses equivalence in multiplication and division. In each lesson, students are asked to find equivalent multiplication or division expressions (number sentences). This may seem like an obscure and unnecessary skill. However, equivalence is a concept that is related to fractions, algebra, and proportional reasoning. The thinking required from students in these lessons, which includes generating many equivalent representations, will help them reason about fractions, algebra, and proportional reasoning. It will also prepare them for more formal proofs that they will be expected to generate and understanding in later years.

Directions: Read the Teacher Note, Reasoning and Proof in Mathematics, on TG pages 121-124 and the Dialogue Box, $6 \times 9 = 3 \times 18$, on TG pages 144-145. Follow the directions for sessions 1.3. As you observe students during these three lessons, ask “in-the-moment” questions and use your assessment checklist to record evidence of students’ ability or lack of ability to create equivalent multiplication expressions and make representations to support their thinking. After class, analyze student papers of students you did not observe and add to the checklist.

Learning Goals:

1. Generate equivalent multiplication expressions by doubling (or tripling) one factor and dividing the other by 2 (or 3)
2. Explain why doubling one factor in a multiplication expression and dividing the other by 2 results in an equivalent expression (benchmark 1).
3. Use story contexts and representations to support explanations.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|--|
| Student has trouble thinking of a story problem and/or a representation to show that $6 \times 9 = 3 \times 18$ or $2 \times 9 = 6 \times 3$ (problems from session 1.1 and 1.2.). | <p>What if there are 6 teams and they each have 9 players? How many people are there altogether? How can these teams change to represent 3×18?</p> <p>Can you draw a picture of the first story problem, 6 teams with 9 players?</p> <p>Now what if there were only 3 teams, but the same amount of players? Do you know how many players would be on each team? Why? Can you draw a picture that shows that?</p> | <p>There is not much time for pulling small groups in these lessons. Monitor students who seem to be having difficulty, ask the “in-the-moment” questions on the left. Pair students who are having difficulty with students who are a little better able to generate equivalent expressions and explain their thinking.</p> <p>Include these students in whole-class discussions by having them present their work if it is mostly correct and having students “turn and talk” with a partner frequently during class discussions to keep all students involved with the ideas.</p> |
| Student has difficulty verbally explaining why doubling and | Do you think halving and doubling always works or just sometimes? | If the student can consistently think of equivalent expressions or can generate several expressions that are |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|---|---|
| halving works or explaining it in writing. | Can you draw a picture or representation to show why halving and doubling works? | equivalent to 40×32 , do not worry too much if the student cannot explain his/her thinking. Pair these students with students who are a little better at explaining their thinking, and highlight clear explanations in class discussions. |
| Student easily generates equivalent expressions using halving and doubling and tripling and “thirthing” and clearly explains his/her thinking using story contexts, representations, and words. | Can you think of another story context or another representation that would also show why this works? | <p>Follow the extension suggestions on TG pages 28, 30, and 34.</p> <p>Have them do the daily practice pages, SAB 2, 4, and 8, even if there is not time for the whole class to do them.</p> <p>Give them more open-ended problems like the ones on SAB pages 7 and 9.</p> <p>Ask them to make up their own page of equivalence problems like the ones on SAB page 8.</p> |

Grade 5, Unit 7, Session 2.1 – Checkpoint 2: Multiplication: How Did I Solve It?

Teacher’s Guide, page 51, and SAB, pages 13-14

Rationale: Investigation 2 focuses on efficient strategies for multi-digit multiplication (benchmark 2). Students already worked on developing efficient multiplication strategies in fourth grade and in Unit 1 of fifth grade. The curriculum authors believe most students “should be carrying out multiplication fluently and efficiently” when they begin this unit (TG page 10). However, the curriculum authors recognize that some students will not have met this expectation. This formative assessment checkpoint allows teachers to assess which students still need to develop or refine an efficient strategy for multiplying 2- and 3-digit numbers, so teachers can work with them during this investigation and Investigation 4. The multiplication strategies that are focused on in this unit are:

1. Breaking the numbers apart by addition
2. Changing one number to make an easier problem
3. Creating an equivalent problem
4. U.S. algorithm

By the end of the unit, students should be able to understand and use at least one strategy efficiently.

Directions: Read the Teacher Notes, Multiplication Strategies from Unit 1, on TG pages 161-162), Creating Your Own Multiplication and Division Problems on TG page 126, and Why Study the U.S. Conventional Algorithms? on TG pages 127-129. Also, read Using the Distributive Property on TG pages 17-19. Follow the directions for session 2.1. As students work on SAB pages 13-14, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Solve multiplication problems efficiently (benchmark 2).
2. Identify the first step of a multiplication solution.
3. Complete a multiplication solution that someone else has already begun.
4. Record solutions clearly and concisely.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|--|---|
| Student uses an inefficient strategy like repeated addition or breaks the numbers apart into too many parts to be efficient. | <p>Can you think of a faster way to solve this problem?</p> <p>For a student using repeated addition say:</p> <ul style="list-style-type: none"> • Repeated addition is not an efficient strategy. Do you think you could use the strategy of breaking apart the numbers to solve this problem? How would you start? Do you know what 10 times this number is? <p>Can you put these numbers into a story context? How many groups of what?</p> | <p>Make sure that the multiplication strategies that have been discussed so far in whole-class discussions are clearly recorded (both with and without arrays) on chart paper and posted where all students can see them.</p> <p>Focus on teaching these students the strategy of breaking apart the numbers or helping them use it efficiently by breaking the numbers apart into reasonable sized parts. If necessary, adjust the numbers in the</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|--|---|
| | <p>For students who are breaking the numbers apart into too many parts:</p> <ul style="list-style-type: none"> • Can you think of a way to shorten the number of steps you are using to solve this problem? How would you do that? <p>What part of the problem have you solved and what is left to solve?</p> | <p>problems (including the problems on their homework) to make them smaller and more manageable and/or put the numbers in a story context to help students understand what the numbers mean.</p> <p>Meet with these students in small groups during sessions 2.2 and 2.4, and if needed, Investigation 4. Since struggling students may work at different speeds and struggle with different parts of the problem, we recommend that you gather these students in a small group, but let them all work independently or in pairs as you observe and give help, as needed. Follow the intervention suggestions in the TG, as needed.</p> |
| <p>Student makes a major error in trying to solve the problem, e.g., student breaks apart the numbers in ways that do not make sense.</p> | <p>Can you tell me how you solved this problem?</p> <p>Do you have a way to figure out if your answer is reasonable?</p> <p><i>Give descriptive feedback, explaining the mistake you see. For example, I think you are trying to use the strategy of breaking apart the numbers by place value, but you left out part of the problem. Let’s try using an array (or a story context) to figure out what part you left out.</i></p> <p>Let’s draw an array. What part of the array have you solved so far? What part(s) do you still need to figure out?</p> <p>Or</p> <p>Can you think of a story problem to go with 75×42? What part of the story problem have you solved so far? What part(s) do you still need to solve?</p> <p><i>If the student is still confused try offering a problem with smaller numbers to solve.</i></p> <p>Try solving 35×42.</p> | <p>The “in-the-moment” questions at the left are meant to help the teacher assess the extent of the students’ strengths and misconceptions. Does the student have a way to figure out if his/her answer is reasonable? Can the student detect his/her own error? Can the student use an array or story context to help him/her keep track of the parts of the problem? Can the student accurately solve a problem with smaller numbers?</p> <p>Follow the intervention suggestions in the box above.</p> |
| <p>Student has difficulty completing a multiplication problem, when giving the first step by someone else.</p> | <p>Can you put these numbers into a story context? How many groups of what? Can you draw an array to represent this problem?</p> | <p>Look at these students’ work carefully after class and monitor these students during session 2.2. Notice which strategies these students use accurately and efficiently and which ones they do not understand or use efficiently.</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|--|---|
| | <p>What part of the problem has been solved with this first step, and what is left to solve?</p> <p><i>If you notice that the first step of the solution is the first step of a strategy that this particular student may not understand, you can tell the student to ask his/her partner for help, or you can give the student a different first step, a first step for a strategy you think the student will be successful with.</i></p> | <p>Focus first, on helping these students develop one strategy, probably breaking apart numbers, that they can use accurately and efficiently. Then help them focus on understanding and using another strategy, probably the U.S. algorithm.</p> |
| <p>Student has the correct answer but has not clearly and concisely recorded his/her thinking.</p> | <p>Can you tell me how you solved this problem?</p> <p>Do you think that what you just told me matches what put down on your paper?</p> <p>What is different about what you told me and what you wrote?</p> <p>What could you add or change so that someone could read it and know exactly how you solved the problem?</p> | <p>If many students are not writing clear enough explanations, try this whole class intervention. Identify two or three student papers with clear explanations. If possible, use papers that show different strategies. Show these papers to the class using an overhead projector or Elmo. After you give students time to read each explanation, ask, “What makes this a clear explanation?” “Is there anything else we could add to or change in this explanation to make it even clearer?”</p> <p>Also, continue to model clear recording of solutions on the board and on chart paper.</p> |
| <p>Student solves both problems accurately and efficiently, demonstrating a clear understanding of at least two of the three strategies listed above.</p> | <p>Do you know any other multiplication strategies? Could you create an equivalent problem to solve this problem? Could you use halving and doubling to solve this problem or tripling and “thirding”?</p> | <p>These students may benefit by being paired together to work on these problems. In sessions 2.2 and 2.4, encourage them to use the strategy of creating an equivalent problem. Follow the extension suggestions in the book as needed.</p> |

Grade 5, Unit 7, Session 3.2 – Checkpoint 3: Division Practice

Teacher’s Guide, page 127, and SAB, page 29

Rationale: Investigation 3 focuses on developing efficient strategies for solving multi-digit division problems (benchmark 3) and using concise notation to record work. Students studied division strategies in fourth grade and Unit 1 of fifth grade. However, they may still need to work on efficiency and concise notation. This formative assessment checkpoint allows teachers to assess how close their students are to meeting these expectations and what they need to work on. The division strategies that students have already studied and continue to study in this unit are:

1. Using groups of the divisor
2. Breaking the dividend into parts
3. Making an equivalent problem
4. Solving an easier, related problem and then compensating

Directions: Read the Teacher Notes, Division Strategies, on TG page 125, Division Notation on TG pages 134-135, and Assessment: 701 Divided by 27 on TG pages 136-138, and the Dialogue Box, Naming Division Strategies, on TG pages 148-149. Follow the directions for session 3.2. As students work on SAB page 29, observe and ask “in-the-moment” questions. After class, analyze the student papers more carefully.

Learning Goals:

1. Interpret and solve division problems with 2-digit divisors efficiently (benchmark 3).
2. Make sense of remainders.
3. Use clear and concise notation to record solutions.

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|---|---|--|
| <p>Student does not use an efficient strategy to solve the problems.</p> <p>Student the student may use repeated addition or skip counting, instead of multiplying the divisor by 10 or a multiple of 10 as a first step.</p> | <p>What do the numbers in this problem mean?</p> <p>Do you think the answer to this problem will be more than 10 or less than 10? Why do you think that?</p> <p>If you know that 10 groups of 64 is 640, why don’t you use that as your first step? How could you record that first step? Now, what else do you need to find out?</p> | <p>Monitor these students throughout the rest of the investigation. Make sure that they are attentive and involved during the whole-class discussions of division strategies. Make charts showing the different division strategies and post the charts where students can easily see and refer to them. If these students continue to have difficulty, try adjusting the numbers in their problems to make them smaller, although make the dividend at least 10 times greater than the divisor. Follow the intervention suggestions in the TG, as needed.</p> |
| <p>Student is not dealing with remainders in a way that</p> | <p>Can you explain your answer to me? How do you know that it is correct? Does your answer fit the story context?</p> | <p>Monitor these students throughout the rest of the investigation. Ask these students to share their work with</p> |

| Evidence of Need for Intervention | “In-the-Moment” Questions | Interventions/Activities |
|--|---|--|
| makes sense. | <i>If the student does not catch his/her own mistake, use descriptive feedback to point out what does not make sense about the way the student is using remainders. Ask the student to try fixing the mistake</i> | a partner to make sure that it makes sense before they turn it in. Also, give descriptive feedback as needed and ask the students to correct their mistakes. |
| Student has the correct answer but has not used clear and concise notation. | <p>Can you tell me how you solved this problem?</p> <p>Do you think you used clear and concise notation? What makes you think your work is clear and concise (or not clear and concise)?</p> <p>Can you think of any ways to make your work clearer or more concise?</p> <p><i>Give the student descriptive feedback about what you think is clear and concise about the student’s work, and what you would like him/her to make clearer or more concise.</i></p> | <p>Model very clear recording of division solutions during class discussions. Record students’ strategies clearly on chart paper and post them where students can easily see and refer to them. Repeat the process of having students show their work and asking the class to comment on what makes the work clear and concise and what would make it even clearer and more concise (TG pages 76-77).</p> <p>For the rest of the unit, ask these (or all) students to show their work to a peer to see if it is clear and concise enough before they turn it in.</p> |
| Student quickly and easily solves the problems and records work clearly and concisely. | <p>What division strategies did you use to solve these problems? Do you know any other division strategies? Have you ever tried the strategy of making an equivalent problem?</p> <p>Give the student a problem with numbers that will not result in a remainder, like 744 divided by 24. Ask the student if he/she could try making an equivalent problem to solve that problem.</p> | <p>Throughout this unit, ask these students to try strategies that they have not tried or are not as familiar with. Encourage them to learn to make equivalent problems. This will further develop their number sense and flexibility with numbers.</p> <p>Follow the extension suggestions throughout the rest of the unit.</p> |