An Analysis of Gender Differences on Performance Assessment in Mathematics -- A Follow-Up Study

Liru Zhang, Delaware Department of Education

Linda Wilson, University of Delaware

Jon Manon, University of Delaware

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Abstract

Interest in gender differences in mathematics remains high after more than thirty years of research. A recent study by Fennema and her colleagues (1998) indicated that real gender differences at earlier ages might be masked by similar achievement results. That is, they found that boys and girls used very different strategies to solve problems as early as grades 1-3, with boys using strategies that imply more conceptual understanding. The sample in this study however, was small and the evaluation instrument was an interview protocol, but the findings of this study point to the needs for more research into the types of problems and the strategies used to solve those problems. The overall results of the study by Wilson and Zhang (1998) showed boys dominating with higher means on constructed-response questions at grades 5, 8, and 10 and higher means on multiple-choice items at grades 3, 8, and 10.

To further explore the gender differences in mathematics, the present study was designed to investigate gender differences in problem solving strategies for two extended constructed-response questions in grade 3. A Sorting Guideline was used for sorting student responses based on current research on children's thinking in the pertinent content area and on the variety of strategies apparent in a sample of student work.

The results of analyzing student responses on the Eggs task demonstrate an interesting pattern that more boys than girls used the most sophisticated approach, yet more boys overall were unsuccessful at accomplishing the task. The girls were more likely to use a visual, more concrete approach, and many more girls than boys did not give sufficient explanation for the strategy used to solve the problem. The results from the JellyBeans task show that more boys (11%) than girls (7%) used Level B, one of the sophisticated approaches, to the JellyBeans task. Other than that, no significant gender-specific differences in strategies used to solve this problem. It is reasonable to say that gender differences in strategies used might have been masked by other factors such as item difficulty.

An Analysis of Gender Differences on Performance Assessment in Mathematics -- A Follow-Up Study

Purpose of Study

The present study is to investigate gender differences in mathematics for students in grade 3 in problem solving strategies used for two extended constructed-response questions.

Methodology of Study

Sample of Students A stratified sample of over 300 students, including one-half boys and one-half girls, was randomly selected from the population of the third graders in Delaware public schools. Students' responses to the two extended constructed-response questions were used for analysis in this study.

Assessment Instrument In 1998, the Delaware Student Testing Program (DSTP) in mathematics was administered to all students in grades 3, 5, 8, and 10. The DSTP math assessment for grade 3 that is aligned to the Delaware Content Standards was developed to measure students' conceptual knowledge, procedural knowledge, and knowledge of mathematical processes across core areas such as Number Concepts, Patterns, Algebra, and Functions, Geometry, Probability and Statistics, and Reasoning and Communication. Two extended constructed-response questions from the third grade math assessment were used for this study (Please see Attachment A).

Data Analysis Descriptive statistics and frequency distributions of test scores on the 1998 DSTP math assessment and on the two extended constructed-response questions were reported for all students and for boys and girls in grade 3. The strategies that students used to solve the problems were summarized by gender.

Procedures for Sorting Students' Responses Four experienced math teachers, each of whom is currently teaching mathematics in grade 3, were invited to participate in this study. Their responsibility was to sort students' responses into categories according to the Sorting Guideline (Please see Attachment B). The Sorting Guideline describes the procedures which students applied to solve the math problems. Then, the Diagnostic Rubric was used to determine which strategy each student used to solve the mathematical task. Student responses sorting involved the following four steps:

> *Step One:* The math experts provided a training session for the teachers to better understand the Sorting Guideline;
>
> *Step Two:* Two teachers worked on one mathematical task. They sorted student responses independently first, following the Guideline and recorded the results (Please see Attachment C for a sample of students' responses);

Step Three: The two teachers worked as a group to discuss the category that they assigned to each student's response. If agreement between the two teachers could not be reached, the math expert made the final judgment; and

Step Four: The math experts, one for each question, determined what mathematical model the student used to solve the problem using the Diagnostic Rubric. Five mathematical models were abstracted from the mathematical strategies that students applied to the JellyBeans task and the Eggs task.

Results of Study

Development of the Diagnostic Rubric By analyzing the mathematics in a task, and then examining a sample of student responses, it is possible to build a rubric that is based on models of the solutions abstracted from the procedures that students used. The rubric usually consists of five to six levels of mathematical cognition, though they may not be hierarchical for each task. In choosing the level descriptions for the rubric, the researchers ask themselves the following questions:

What kind of mathematical model is the student using to solve the problem? Sometimes more than one solution strategy makes use of the same mathematical model, so that the term “model” is used more broadly than the term “solution strategy.” In this study, the Sorting Guideline represented the range of solution strategies found in the sample responses, but some of these strategies were then combined to form the Diagnostic Rubric levels, when it was determined that the strategies essentially made use of the same mathematical model.

How complete is the model? Sometimes a student's model will only serve to accomplish the first part of the task, but will not likely lead to a complete solution.

How sophisticated is the abstraction? This determination is usually based on the mathematics in the task.

Diagnostic Rubric for the JellyBeans Task Levels A and B of this rubric represent students who recognize the given data as a sample, which seems to be the mathematical essence of this task. Students in Level A show an ease with using an estimated answer of 20 as a typical number of beans in each cup, which allows them to find a reasonable answer through a simple computation of 10 times 20. At Level B, students seem to feel more comfortable with a more “exact” answer, and assign their own “randomly-generated” data to the remaining cups, then find a total. Level C is a cruder model, but one in which an estimate of the total is still based upon a reckoning of the number of beans in the first four cups. In a Level C response, the student sees the first four cups as a unit and then recognizes that the total for all ten cups is greater than that. Levels A, B, and C, although progressively less sophisticated, might all be described as exemplifying statistical models. Level D responses involve totals (when found) which are seldom unreasonable but the focus of the cognitive processing has shifted to one of finding a pattern in the data rather than thinking of the data as representing a random distribution. The category of "other" responses usually leads to incorrect solutions.

A Used estimation or an average to find the approximate number of beans in each cup then multiplied by 10.

This is an elegant statistical solution in which the student recognizes that each of the labelled cups represents a sample of beans, that a typical value for each of the ten cups will, therefore, be around 20 beans per cup, and then estimates the total as 10 cups times 20 beans per cup or around 200 beans.

B Reasonable assignment of numbers of beans to the rest of the cups.

This approach recognizes that the first four cups represent sample values, but rather than assuming a typical value for each cup as in strategy A, achieves an estimate by assigning similar values of beans to each of the six remaining cups. Thinking in A is characterized by assuming that a single "average" value can be taken for each cup. Thinking in B achieves a similar end by assigning what we take to be values randomly distributed around a typical value. Students using this strategy may or may not remember to sum the beans in all the cups once the "random" assignments are made.

C Added numbers of beans from first four cups, found the sum to be 79, and then extrapolated to a not-unreasonable estimate of the total number of beans.

Students adopting this strategy are using a part-to-whole estimation strategy in which they recognize that a part of the total is represented by the sum of the beans in the first four cups and that the total is, therefore, somewhat larger than this sum. Rather than treating one cup as the unit of analysis, students using this strategy treat the four cups to which a number of beans are assigned as the unit. Typically, they do not reason in a strictly proportional manner, i.e. do not treat the 79 as 40% of the total. Rather, they know that the total must be somewhat larger than 79.

D Interpreted the given numbers on the cups to be the start of a pattern, and continued that pattern in assigning numbers to the remaining cups. May or may not have computed a sum for all ten cups.

Clearly the focus of the task for these students is to continue the pattern they perceive in the first four numbers, i.e. 19, 21, 21, 18. They have cognitively characterized this task as a "find-the-pattern task" rather than as an estimation task. Students exhibiting this type of response have misread the intent of the task and missed the statistical aspect of the task altogether as they regard the rules of assignment to be patterned rather than random.

O Other responses

These responses include a broad range of possible responses including, but not limited to, unreasonable assignment of values to the remaining cups, summing of the first four values without attempting an estimate of the whole, expressing the idea that one can't know how many beans are in the jar or simply offering as a solution a "guess" or a "lucky number" or no response. In many instances, these responses seem to represent a visual or logical consideration of the number of beans in the large jar without any reference whatsoever to the samples in the cups.

Diagnostic Rubric for the Eggs Task The levels of this rubric are collapsed into the following categories, which are considered to represent different cognitive levels. Level A represents the most mathematically sophisticated level. Level B, more simplistic and more visual in its orientation, accomplishes the task working from what might be characterized as an essentially manipulative model. These two approaches constitute the vast majority of student responses that were successful at the task. At Level C, the response is too brief to enable reviewers to determine the strategy used. Level D and "other" represent incorrect responses.

A Found the total number of eggs (this may be implied rather than apparent), then used repeated subtraction, or division, or repeated groupings of twelve to find number of cartons.

At this level, the student shows an ability to model the problem mathematically and use operations on whole numbers to accomplish the task. This is a numerical solution to the problem, in which the student first finds the total number of eggs, and then figures out how any groups of twelve there are in 39. This could be done by subtracting 12 from 39 repeatedly, by dividing 39 by 12, or by adding or multiplying groups of 12 until 39 is reached.

B Used groupings of twelve without finding the total number of eggs (perhaps marked groups of twelve on the diagram).

This is a more visual approach to the problem. Some students simply marked off every twelve eggs on the diagram, until all eggs are accounted for. Others drew a new diagram, as if they were placing the eggs into cartons of twelve. Since the eggs are put into groups of twelve until all are gone, it is not necessary to compute the total number of eggs. Without a clear appeal to number, the task is accomplished in a concrete manner.

C Said 4 cartons, or 3 cartons with 3 left over, but gave incomplete explanation, or showed no evidence of strategy.

These responses did not give sufficient evidence to determine which strategy was used. There is a difference here between those who can see that a fourth carton *needs to be used, even if it is not full, and those who are unwilling to put the leftover eggs into another carton.*

D **Added to get total number of eggs, but then did not attempt to find number of cartons.**

In these responses, the student was not clear on how to accomplish the task, but began by finding the total number of eggs and then went no further.

O **Added, or other computations with the number in the problem, without regard for answering the question, guessed, off task, or no response.**

Some students had no idea how to accomplish the task, and so either left it blank, wrote something off task, or simply computed a sum with the numbers in the problem.

Results of Data Analysis Means, standard deviations, and frequency distributions of raw scores on the 1998 DSTP math assessment are reported for all students and by gender in Table 1. The results indicate that no significant difference in achievement in mathematics has been found between the gender groups for students in grade 3. Boys and girls received a very similar mean score, 45.7 for boys and 46.0 for girls. Based on the item level analysis, boys and girls demonstrated a very similar pattern of frequency distribution of item scores on the JellyBeans task with almost equal mean scores for both groups (mean=1.42 for boys; mean=1.45 for girls). More girls (19.5%) than boys (15.9%) received the highest item score (4-point) on the Eggs task, and nearly equal percentages of the boys (33.5%) and girls (33.6%) received the second highest item score (3-point). Eighteen percent of the boys did not solve the problem at all compared with 16% of the girls who fell into the same category. Although the mean item score for girls (mean=2.31) is higher on the Eggs task than that for boys (mean=2.23), no statistically significant gender difference is suggested according the result of a t-test ($t=1.03$; $p<.41$). The item mean scores, however, suggest that the JellyBeans item (mean=1.44) is more difficult than the Eggs item (mean=2.27).

The strategies that students used for problem solving are summarized in Tables 4 and 5. For the JellyBeans task (Table 4), only 3% of the boys and the girls used Strategy A. More boys (11%) than girls (7%) used Strategy B, but more girls (22%) than boys (21%) used strategy D. A very small number of students used Strategy C (3% for boys; 2% for girls), whereas a quite large percent of students (62% for boys; 66% for girls) used other strategies that led to the incorrect solution to the problem.

For the Eggs task (Table 5), more boys (17%) than girls (13%) used strategy A, but more girls than boys used strategies B (28% for boys; 34% for girls), C (18% for boys; 21% for girls), and D (4% for boys; 7% for girls). Thirty-three percent of the boys and 25% of the girls had no idea how to solve the problem.

Discussion

<u>Discussion on the JellyBeans Task</u> The same percentage of boys and girls (3%), only 5 student for each group, used Level A to solve the JellyBeans task. More boys (11%) than girls (7%) used the approach of Level B for the task. Although both Levels A and B of this rubric represent students who recognized the given data as a sample, students used different strategies to solve the problem. Level A students used an elegant statistical solution in which students estimated the answer of 20 as a typical number of beans in each cup, then estimated the total, whereas Level B students looked for the "exact" answer by assigning their own "randomly-generated" data to the remaining cups, then find a total. Both strategies can be considered based on a statistical/mathematical approach.

For the level C responses, students used a part-to-whole estimation strategy in which they recognized that a part of the total is represented by the sum of the beans in the first four cups as a unit and that the total is, therefore, somewhat larger than the part. This is a direct, but rather simplistic approach. Only six students used this strategy out of the whole group of 305 students. It seems that students in this Level tried to use a statistical approach to solve the problem, however, Level C is less mathematically sophisticated than Levels A and B.

In the sample 22% of the boys and 23% of the girls used Level D. Those students characterized this task as a "find-the-pattern task" rather than as an estimation task. They might have misread the intent of the task or missed the statistical aspect of the task or been unable to apply a sophisticated mathematical procedure to solve the problem. The considerable size of this group may be an indicator of the fact that instruction in pattern finding is increasingly prevalent in the early grades.

It is worth noting that over one half of the students (62% for boys; 66% for girls) could not accomplish the task. This group used "other" strategies or offered unreasonable estimates or failed even to attempt the task. The item statistics show that this is a more difficult task. The average score for this item is 1.44 out of 4.00 for all students. In terms of item scores, nearly equal percentages of the boys and girls received the same score. For example, 11-12% of the boys and girls had a score of 4; 11% of them had a score of 3; 12% of the students had a score of 3; 35-36% of the boys and girls had a score of 2; and 26-27% of them had a score of 0.

More boys (11%) than girls (7%) used Level B, one of the sophisticated approaches, to the problem. Other than that, no significant gender-specific differences in strategies used to solve the JellyBean task. Is this task too difficult for many third graders? Or is it true that there are no gender differences in problem solving strategies? Or is the gender difference masked by the difficulty of the task? It seems to be unreasonable to exclude the possibility of gender differences in problem solving strategies because other factors such as item difficulty might have covered such differences.

Discussion on the Eggs Task Of the students who gave sufficient evidence of their strategies to the Eggs task, a significant difference shows up between Level A and Level B on the Diagnostic Rubric. More boys (17%) used the approach of Level A, which is a more mathematically sophisticated approach. At this level, students show an ability to use two steps of operations with whole numbers. First, the student finds the total number of eggs by addition, and then determines the number of cartons by division, repeated subtraction, multiplication, or repeated addition. In any case, the task is to find the number of 12s in 39. Level B, where we found more girls' responses (34%), represents a more practical approach to the problem. Students either circled groups of twelve on the drawing, marked the eggs as they counted groups of twelve, or drew new cartons of eggs and visually "placed" the eggs into cartons until all were accounted for. While the task is accomplished, one could argue that this approach is not as abstract as Level A, and there is no evidence given of an understanding of mathematical operations on whole numbers. If the number of eggs were greater, one can see that this approach to the problem would be cumbersome and tedious, whereas the Level A approach would work efficiently for any number of eggs. So, in terms of applicability of the approach to more complex problems, it is perhaps troubling to see girls using a less sophisticated approach. The approach that more girls used (B) was more visual and concrete, but would be cumbersome to use with larger numbers.

For the Level C responses it was impossible to determine the exact strategy used, whether it might have been that of Level A or B or some other strategy. These students got a correct answer (4 cartons) or a nearly correct answer (3 cartons with 3 left over), but did not adequately explain how they got it. More girls' (21%) than boys' responses (18%) fell into this category, leaving us with questions about why this might be. Were the girls less able than the boys to explain their thinking? If so, is that because their metacognitive skills are less well developed, or is it that they were less able to put into words or a drawing what they had done? In either case, the girls were less able (or willing) to communicate how they approached the problem. It could be the case that the student who did not leave evidence performed some sort of counting strategy, where they perhaps placed their pencil on each egg and mentally counted to 12 until all eggs were accounted for (meaning that, had they given evidence, their responses would have been sorted into Level B rather than Level A). It seems less likely that a student who wrote nothing but the answer was able to mentally calculate 21 + 18 and then how many 12s are in 39. If that is the case, it would lend more credence to the gender difference between A and B, since more girls fell into category C. Of course, given only the written responses, it is not possible to say for certain.

There were only 17 students out of 300 who began the problem by finding the total number of eggs, and then stopped there. Of those, almost twice as many were girls (7%) as boys (4%). Though the numbers are small, this could be an indication that girls were more likely to want to perform some type of algorithm to accomplish the task, and adding the numbers of eggs (the only numbers in the problem) seemed to be the most accessible. Taken together with the responses that fell into the "other" category, more boys (33%) than girls (25%) were unable to accomplish the task. The boys were more likely to leave it blank, write something off task, or do some meaningless computation with the numbers in the problem, such as adding 21, 18, and 12.

It is interesting that more boys than girls used the most sophisticated approach to the problem, yet more boys overall were unsuccessful at accomplishing the task. The girls were more likely to use a visual, more concrete approach, and many more girls than boys did not give sufficient explanation for the strategy used to solve the problem.

Limitations of Study

The primary purpose of this study was to investigate the gender differences in strategy use to solve two extended constructed-response questions for students in grade 3. Other factors, such as number of tasks, item type, item difficulty, and student achievement in mathematics might limit the results of the present study. In spite of the limitations, the findings encourage more research studies to further explore gender differences in strategies used to solve mathematical problems across grades.

Table 1

Student Performance
on 1998 Math Assessment by Gender

Statistics	**Boys**	**Girls**	**Total**
Mean	45.7	46	45.8
S. D.	14.1	13.7	13.9
N	4100	3842	7972

Table 2
Item Statistics for JellyBeans Task

Item Score	**Boys**		**Girls**		**Total**	
	N	%	N	%	N	%
4	450	(11.0)	471	(12.2)	921	(11.6)
3	458	(11.2)	413	(10.7)	871	(10.9)
2	503	(12.3)	482	(12.4)	985	(12.4)
1	1438	(35.1)	1403	(36.2)	2841	(35.6)
0	1105	(27.0)	1006	(26.0)	2111	(26.5)
Off Topic	146	(3.5)	97	(2.5)	243	(3.0)
Mean	1.42		1.45		1.44	
SD	1.31		1.32		1.32	
N	4100		3872		7972	
T-Test	$t=1.02$ ($p<.576$)					

Table 3
Item Statistics for Eggs Task

Item Score	Boys	Girls	Total
	N %	N %	N %
4	650 (15.9)	754 (19.5)	1404 (17.6)
3	1373 (33.5)	1302 (33.6)	2675 (33.6)
2	838 (20.4)	642 (16.6)	1480 (18.6)
1	499 (12.2)	581 (15.0)	1080 (13.5)
0	622 (15.2)	520 (13.4)	1142 (14.3)
Off Topic	118 (2.9)	73 (1.9)	191 (2.4)
Mean	2.23	2.31	2.27
SD	1.3	1.32	1.31
N	4100	3872	7972
T-Test	t=1.03 (p<.405)		

Table 4

Strategies Used by Gender for JellyBeans Task

Strategy	**Boys**	**Girls**	**Sub-Sum**
	N %	N %	N %
A	5 (.03)	5 (.03)	10 (.04)
B	17 (.11)	11 (.07)	28 (.10)
C	4 (.03)	2 (.01)	6 (.01)
D	32 (.21)	34 (.23)	66 (.21)
O	93 (.62)	102 (.66)	195 (.64)
Sum	151 (.50)	154 (.50)	305

Table 5

Strategies Used by Gender for Eggs Task

Strategy	**Boys**	**Girls**	**Sub-Sum**
	N %	N %	N %
A	25 (.17)	20 (.13)	45 (.15)
B	41 (.28)	52 (.34)	93 (.31)
C	27 (.18)	31 (.21)	58 (.19)
D	6 (.04)	11 (.07)	17 (.06)
O	50 (.33)	37 (.25)	87 (.29)
Sum	149 (.50)	151 (.50)	300

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Attachment A

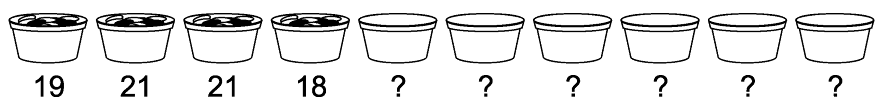
Extended Constructed-Response Questions and Scoring Rubrics

Standard Measured:

Standard 9: Students will develop an understanding of STATISTICS AND PROBABILITY by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions; to present convincing arguments; and to model mathematical situations to determine the probability.

Item:

Ms. Mars has a big jar filled with jelly beans. You must estimate how many beans are in the jar. As a hint, Ms. Mars poured out the beans and they filled 10 cups. She counted 19 beans, 21 beans, 21 beans, and 18 beans in the first 4 cups.



What is your estimate for the total number of beans in the big jar? Explain how you got that number.

Scoring Rubric:

4 Prediction from 190 – 210 with complete mathematical explanation. This might include a well-reasoned assignment of sample values (perhaps of 18,19, 20 or, even more likely, 21) to each of the remaining six cups and then an addition of these simulated values. Or the child might pick an average number in each cup (this might be 18, 19, 20, or 21) and then multiply by 10. Explanation need not discuss "average" to constitute a valid prediction.

3 Prediction should be from 180 – 220 but explanation not as complete as required to receive a 4. For example, a child might assign values to each of the six mystery cups but not discuss why these assignments are reasonable.

2 Prediction of 150 – 250 but explanation weak or missing.

1 Skewed prediction (obviously too low or too high) with weak or missing explanation.

0 Blank/no response.

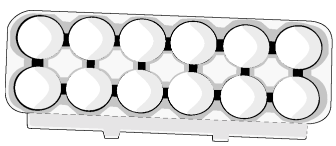
Commentary:

This item addresses estimating the number in a population based upon the sizes of several samples from that population. Very mature responses noted that each of the given cups contained approximately 20 beans so a reasonable estimate for the entire jar would be 10x20=200 beans. More typically, students assigned reasonable estimates to each of the six remaining cups and then found the total by summing the four given and six assigned values.

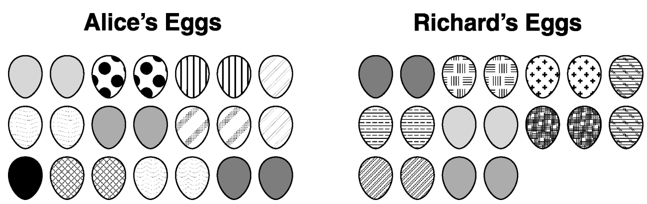
Standard Measured:

Standard 6: Students will develop NUMBER SENSE by solving problems in which there is a need to represent and model real numbers verbally, physically, and symbolically; to use operations with understanding; to explain the relationships between numbers; to apply the concept of a unit; and to determine the relative magnitude of real numbers.

Item:



Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



How many cartons will they need to hold all the eggs? Explain how you got your answer.

Scoring Rubric:

4 Correct answer (4 cartons) with clear verbal or pictorial explanation. (This may or may not involve finding the total number of eggs. For example, the problem might be successfully solved by grouping the egg images into groups of 12.)

3 Correct answer (4 cartons) but sketchy explanation or incorrect answer of 3 cartons with explanation that describes the number of cartons *filled*.

2 There is an (unsuccessful) attempt to divide eggs into cartons. Perhaps total number of eggs (39) is correct but attempt to divide into cartons is flawed.

1 An attempt is made to count eggs (perhaps even successfully), but there is no evidence of an attempt to divide eggs into cartons.

0 Trace evidence of work but without clear connection to problem situation.

Commentary:

This item addresses several components of number sense. The student must use several operations with understanding including addition and division but is able to do this in a context involving a physical representation of number. A great variety of solution strategies have been observed. For example, some students actually identified the first, second, and third dozen eggs in the diagram and found that three full cartons were needed and a fourth with only three eggs. Other students found the sum directly and compared this to the number of eggs in three dozen.

Attachment B

Sorting Guideline for JellyBeans Task

A Used estimation or an average to find the approximate number of beans in each cup, then multiplied by 10.

B Randomly assigned reasonable number to the remaining cups, then added. May or may not have shown evidence of recognizing that the sum is an estimate.

B1 Randomly assigned reasonable numbers to the remaining cups. Did not find a sum or answer incorrect.

C Interpreted the given numbers on the cups to be the start of a pattern, and continued that pattern in assigning numbers to the remaining cups then either did or did not get a sum.

D Assigned number to cups that are not within a reasonable range. May or may not have computed a sum.

E Ignored the given information on the cups of beans and made an estimate of the total without explanation, perhaps by studying the picture of the jar.

F Expressed the idea that one “can't know” the number in the jar, so made a wild guess or used a “lucky number.”

G Simply added first four numbers (sum of 79).

G1 Added the first four numbers but then extrapolated to estimate sum of all ten cups.

O Others

N Response is off task.

N1 No response

Sorting Guideline for Eggs Task

A Found the total number of eggs, then used repeated subtraction or division or repeated groupings of twelve to find number of cartons.

B Used groupings of twelve without finding the total number of eggs.

C Added to get the total number of eggs, but then did not attempt to find number of cartons.

D Added, or did other computations with, the numbers in the problem, without regard for answering the question.

E Gave estimated answer with no mathematical support (e.g., guess).

N Response is off task or no answer.

O Other

F Says 4 cartons or 3 cartons with 3 left over but gives no evidence of a strategy.

G Says 3 cartons with 3 left over but gives no evidence of a strategy.

Attachment C

Sample of Student Responses to the JellyBeans Task

224

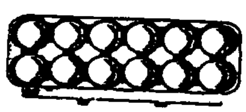
65

Ms. Mars has a big jar filled with jelly beans. You must estimate how many beans are in the jar. As a hint, Ms. Mars poured out the beans and they filled 10 cups. She counted 19 beans, 21 beans, 21 beans, and 18 beans in the first 4 cups.

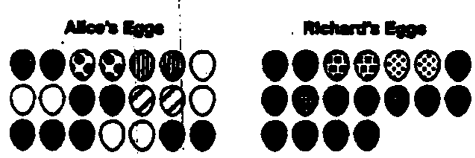
What is your estimate for the total number of beans in the big jar? Explain how you got that number. 200

I got that number because I figured about all the numbers will ~~estimate~~ to about 20. Then I added.

66



Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)

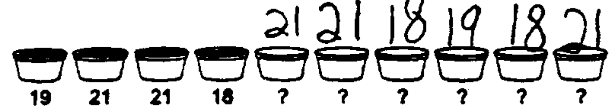


How many cartons will they need to hold all the eggs? Explain how you got your answer.

4 ca You would need 4 cartons of eggs and you would have 9 left over. / I got my ancer by adding how many eggs they dideint saw how many times 12 could go into 39 (wich is the sum of 21+18) then I saw how many were left over.

65

Ms. Mars has a big jar filled with jelly beans. You must estimate how many beans are in the jar. As a hint, Ms. Mars poured out the beans and they filled 10 cups. She counted 19 beans, 21 beans, 21 beans, and 18 beans in the first 4 cups.

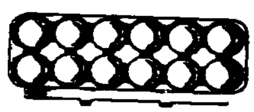


What is your estimate for the total number of beans in the big jar? Explain how you got that number.

197

I estimated.

66



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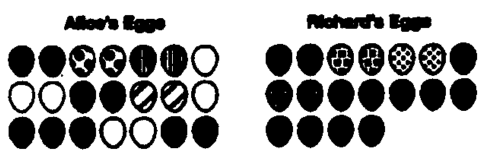
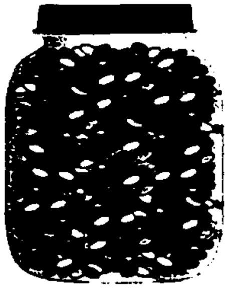


How many cartons will they need to hold all the eggs? Explain how you got your answer.

3 1/2. I conted how much carton I would need.

GO ON

190

65 

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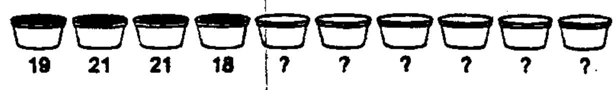
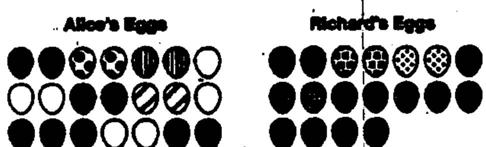


What is your estimate for the total number of beans in the big jar? Explain how you got that number.

150 I got that number because I added up 19,21,21,18 =79 so I just guest and got 150

66 

Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



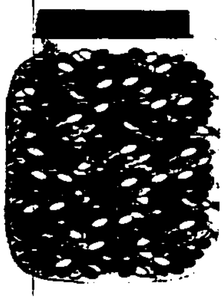
How many cartons will they need to hold all the eggs? Explain how you got your answer.

3 cartons I got that ansver because I counted the eggs until I got 12 so thats 1 carton and I did the same until I got 3 cartons and I had two eggs left

GO ON

121

65



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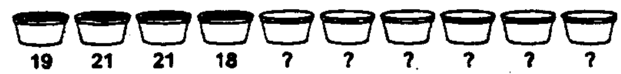
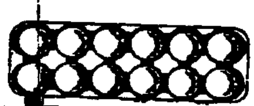


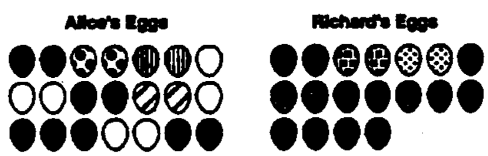
What is your estimate for the total number of beans in the big jar? Explain how you got that number. 208 jelly beans

Because I added numbers in this pattern: 19, 21, 21, 18, 19, 21, 21, 18.

66



Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



How many cartons will they need to hold all the eggs? Explain how you got your answer.

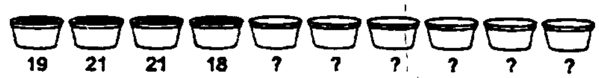
4 cartons

First I added 21 and 18. Then I devided 12)39.

GO ON

65

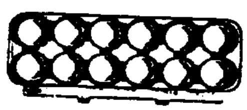
Ms. Mars has a big jar filled with jelly beans. You must estimate how many beans are in the jar. As a hint, Ms. Mars poured out the beans and they filled 10 cups. She counted 19 beans, 21 beans, 21 beans, and 18 beans in the first 4 cups.



What is your estimate for the total number of beans in the big jar? Explain how you got that number.

79 is my estimation because If you add 19+21+21+18 it will equal 79.

66



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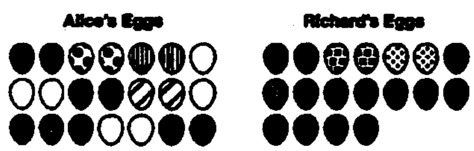


How many cartons will they need to hold all the eggs? Explain how you got your answer.

They will need 4 cartons for ttting 48 eggs.

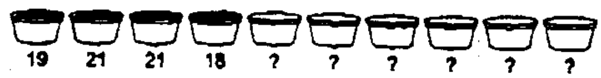
GO ON

Attachment C

Sample of Student Responses to the Eggs Task

65

Ms. Mars has a big jar filled with jelly beans. You must estimate how many beans are in the jar. As a hint, Ms. Mars poured out the beans and they filled 10 cups. She counted 19 beans, 21 beans, 21 beans, and 18 beans in the first 4 cups.



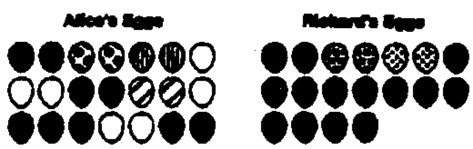
What is your estimate for the total number of beans in the big jar? Explain how you got that number.

I got 198 because on my calculator I continued the pattern that Ms. Mars already started.

66

Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)

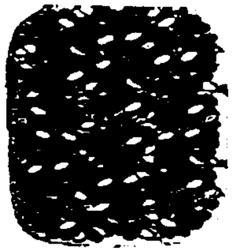
A level



How many cartons will they need to hold all the eggs? Explain how you got your answer.

First I added 21 and 18. Then I thought of my 12 times tables. 36 is the closest number to 39 (my answer so far). It would take about four cartons.

GO ON



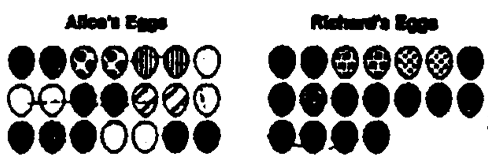
out the beans and they filled 10 cups. She counted 19 beans, 21 beans, 21 beans, and 18 beans in the first 4 cups.

What is your estimate for the total number of beans in the big jar? Explain how you got that number.

5522 if you
21's put 3 more
and 2 more
18's and 3 19 and
then add up you would get 5522.

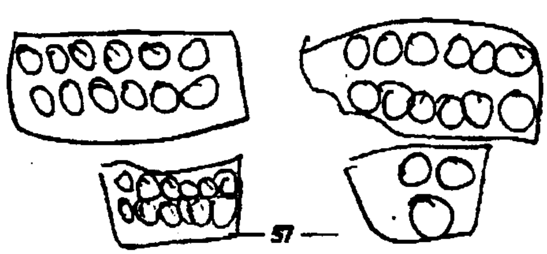
66

Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



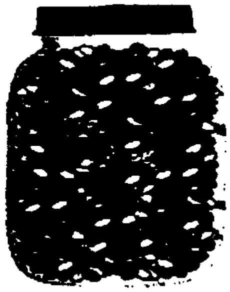
How many cartons will they need to hold all the eggs? Explain how you got your answer.

4 dozens



B level

65



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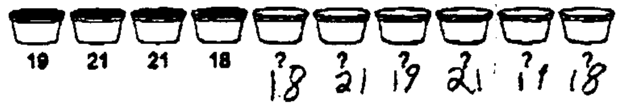
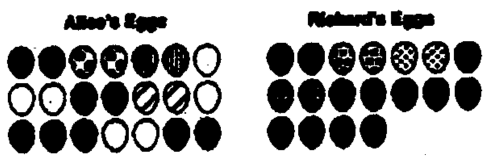


What is your estimate for the total number of beans in the big jar? Explain how you got that number.

I got the anwser because I estamated and got 19, 21, 19, 21, 19, 18.

66

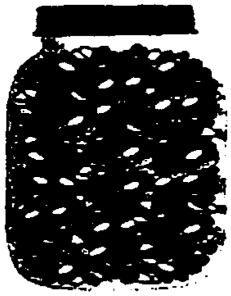
Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



How many cartons will they need to hold all the eggs? Explain how you got your answer.

3 egg cortons and a bag with 3 eggs in it.

C level



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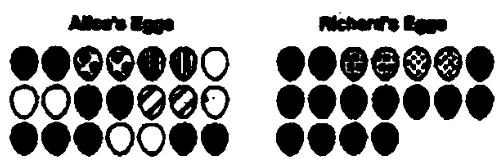
What is your estimate for the total number of beans in the big jar? Explain how you got that number.

The answer is 79 because I added 18+19+21+21=79.

Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



How many cartons will they need to hold all the eggs? Explain how you got your answer.

The answer is 39 because I added 21+ 18=39 cartons.

D level